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SIMULACHRON
A simulation model for a combined heat and power production system

Helge V. Larsen

Abstract: This report gives a short survey of problems concerning the evaluation of development plans for a combined heat and power system. A rather detailed model describing technical and economic aspects of the system is set up. This comprises condensing, extraction, and back-pressure power plants, district heating boilers, and day-to-day heat storage facilities. Transmission systems for heat and power are also modelled. The model has been implemented on a computer. By means of the computer program, called Simulachron, the operation of a combined heat and power system can be simulated on an hour-to-hour basis. Test-runs of the program covering 48 time steps and some 30 power plants are documented.

EDB Descriptors: COGENERATION; COMPUTERIZED SIMULATION; COST; DISTRICT HEATING; DUAL-PURPOSE POWER PLANTS; HEAT; HEAT STORAGE; OPERATION; OPTIMIZATION; PLANNING; POWER GENERATION; PRODUCTION; S CODES

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APPENDICES:

PREFACE

This thesis is submitted as part of the requirements for the technical licentiate degree (Ph.D.) at the Technical University of Denmark. The study has been carried out at Risø National Laboratory in close cooperation with professor J. Rich. Hansen at the Electric Power Engineering Department of the Technical University.

The principal aim of the research has been to produce a detailed mathematical model of a combined heat and power production system. This model considers technical and economic aspects of condensing, extraction, and back-pressure power plants, district heating boilers, and day-to-day heat storage facilities. To simulate the hour-by-hour operation of the system the model has been implemented on a computer. Test-runs of the computer program, called Simulachron, covering 48 time steps and some 30 plants are documented.

1. INTRODUCTION

1.1. Presentation of the problems

The two main sectors of the Danish energy systems that will be dealt with in this thesis are the supply systems for electricity and space heating. The electricity has during several decades mainly been produced in centralized power plants and distributed collectively, whereas the major part of the demand for space heating earlier was - and to a great extent still is - satisfied by individual production units (boilers in single- and multi-family houses). The increased use of district heating has collectivized almost half of the market for space heating. Also, the introduction of electrical heating and of natural gas (methane) for private boilers will increase the collective nature of the system.

Because of the great capital costs in collective systems their development calls for a thorough analysis of competing scenarios. Very often computers prove to be valuable in these analyses to simulate the future operation of the system for several years. In some cases, especially when the system considered satisfies only one type of demand, the computations need not be very detailed. For instance, when building a new district heating system, or when expanding an already existing system, there is no need for any detailed computer simulations of the operation of the system as far as heat-only (i.e. no CHP*) production) systems are concerned. Naturally, there might be questions concerning the profitability of the project. But the results from rather simple computations based on load duration curves, for instance, could give all the information needed to compare different schemes.

*) CHP = combined heat and power

When turning to electric power production the situation is somewhat different. Because of the great complexity and size of the power system one must use rather detailed simulation models when development plans for the production system are evaluated and compared with regard to operational flexibility and production economy. Such models have been used for a few decades. CHP production has to some extent been incorporated in the models.

It is not only the capital cost and the cost of production (mainly fuel costs) that is focussed on, when development plans are evaluated. The "quality" of the electricity supply should be taken into account, too. In this context "quality" covers such fields as correct frequency and voltage (magnitude and waveform), and security of supply. To estimate the voltage and frequency deviations that might arise, the simulation of the production and transmission system and loads must incorporate time steps of some milliseconds. The security of supply, often described by LOLP (loss of load probability), can be found only by introducing the reliability of each power-producing unit, and matching the total expected available capacity of production with the expected daily maximum loads. This subject, the "quality" of the supply of electricity, will not be considered further in this report.

The introduction of CHP has caused a quite new situation in the evaluation of development plans. This stems from the requirement that the production units must simultaneously satisfy the demands for heat and power. The units must be able to follow demands that vary incoherently during the day and year. In order to minimize total operating costs heat production at the CHP plants is given priority over power production. This means that at any given moment heat production might impose constraints on the latter (the minimum and maximum power production attainable depends on the heat production). As a whole, at each moment the normal operation of the CHP system is aimed at satisfying the heat demands in the individual district heating areas supplied from CHP plants, and the power demand in the overall area, all at minimum cost.

An increased use of CHP in the Danish power system will give rise to a new operational problem that already has been experienced in local areas. During times of heavy heat loads, but minor power loads (a cold winter night), the minimum power production from the CHP plants might exceed the power demand*). In such a case there are only a few actions that can be taken: Firstly, one must try to sell the excess power to some neighbouring power system, otherwise one must bypass the turbines at some plants and feed the steam from the boilers directly to the heat exchangers for district heating. Alternatively, a CHP plant can be taken out of operation, and the heat production moved to a district heating boiler, normally used as a peak-load boiler. Finally, possible heat storage facilities could be discharged at these critical times.

The difficulty just mentioned would be increased by the introduction of nuclear power plants as these usually are operated as base-load plants producing almost constant power for reasons of economy. Also wind-generated power could increase the problems.

The use of CHP will increase considerably in the future. This can be seen from the following: In 1981 42% of the demand for space heating in Denmark was supplied as district heating, and 40% of this was produced in combined production. According to plans (ref. 15) these figures will be 51% and 63% by 2000, respectively. In same plans the demands for electricity and space heating during the same time period are expected to increase by 55% and 19%, respectively. From this it follows that the demand for co-produced heat will increase by 47% relative to that for the electricity. This shows that the problems in operating the CHP system will increase during the years to come.

Not only the growing amount of CHP production, but also the introduction of energy production systems (e.g. wind turbines and

*) The minimum power production from a CHP plant is approximately proportional to the heat production (see Sections 2.3 and 2.4).

solar panels for both heat and power) that cannot easily be regulated in the same way as thermal plants, accentuates the need for more detailed developmental analysis in future planning. If, for instance, 10% of the electricity (energy) should be produced by wind turbines, the installed capacity of these should amount to about 17% of the maximum power demand (ref. 17). This is due to the low utilization time for wind power plants compared to normal plants. Two aspects of such renewable energy sources should be considered: the fuel savings and the capacity displacement. None of these two figures can be found generally, as they both depend on the way in which the total system is actually operated. The capacity displacement is especially difficult to estimate.

From the previous paragraphs of this section it can be seen that there is an increasing need for more detailed analysis of development plans for the CHP production system in particular. In this analysis one needs a prognosis for the demands for heat and power. But such forecasts can be given only with a certain accuracy. Therefore development planning consists of analyses and a comparison of several scenarios. Computer simulations are inevitable in such work. These simulations should include as many as possible of the previously mentioned aspects of the operation of a CHP system.

Such simulations are also useful in evaluating single technologies (e.g. wind power, heat storage, and electrical storage heating), both from economical and operational points of view.

There are several types of problems that must be solved when planning and operating a CHP system. The time scales involved in these problems are very different, ranging from minutes for the operation of the system to years for long-range development planning. Until now this chapter has dealt mainly with the latter planning problem, but in the following paragraphs this will be supplemented by a short description of other such problems as listed in Table 1.1.

Table 1.1. Planning problems for a CHP system.

Planning problem	Time horizon
Development of production system	20-30 years
Development of transmission system (heat and power)	5-15 years
Maintenance scheduling	2-5 years
Fuel purchase planning	1-5 years
Unit commitment	1-7 days
Scheduling the operation of day-to-day heat storage facilities	1-3 days
Load dispatching (heat and power)	continuously

One of the more important foundations for development plans for the CHP production system are a range of forecasts for the heat and power demand over the next 20 or 30 years. Such prognoses should give the yearly energy demands and maximum loads. From this - and from data describing the existing and the candidate future CHP plants - the overall aspects of development plans can be evaluated.

Also, the expansion of the heat and power transmission systems is a task that calls for thorough planning. Although the time horizons normally are shorter than mentioned above, it is still necessary to have prognoses for the heat and power demands for some years ahead. These forecasts must also contain a roughly specified spatial distribution of the loads.

Another important object of planning is the maintenance scheduling for the next few years. This calls for a recording of existing inter-utility or inter-state contracts - and an estimate of possible new contracts. In this case the prognosis for the heat and power demands must be more detailed, in that the yearly load variations must also be forecast. The aim of the maintenance scheduling then is - with due regard to the production costs - to place in time the yearly routine maintenance for the individual production units in such a way that the security of supply never becomes too low.

Also the purchase of fuel must be planned for some years ahead. In this planning an analysis of fuel and fuel transport markets must be carried out together with an estimate of the future need for fuel at each power plant site.

When planning the daily operation of the heat and power production system one must have short time forecasts for the hourly heat and power demands for the next few days. For the first, say 24 hours ahead, the forecasts should be rather accurate. Such short time load variations are, to a great extent, derived from experience gathered through several years of operation of the system - but naturally the weather forecasts also play an essential role for these prognoses.

From these short-time estimates of the loads it is decided in advance when each generator is to be started and shut down. Depending on the type of power plant, a decision to start a plant must be taken some time before the plant is actually planned to be operated. This time ranges from a few minutes for peak-load plants to half a day for base-load plants. The aim of this "unit commitment" is to perform an economically optimal trade-off between costs associated with the starting of the plants and the costs of running the plants, mainly those of fuel. Many constraints are imposed on this unit commitment. For instance, it should be checked that no power transmission line would be overloaded.

As an integral part of the system the inclusion of possible heat storage in connection with CHP production should be considered, too.

Thus, having established a plan for which generators are operated at each moment, the remaining task consists of dispatching the load (heat and power) among these generators, while paying correct attention to the operation of the storage facility. The object of this load dispatching is to minimize the total running costs of the production system.

Computer programs to solve these problems concerning the planning and operation of power systems have been available for many years. In the sixties the procedures were rather simple, but during the following years the computers became cheaper and much more powerful in terms of size of their memory and speed of computation. In accordance with this, very detailed simulations and optimizations can now be performed. The need for such production simulations is intensified by the greater relative role played by fuel costs at present (except in the case of nuclear power) than in the past.

In the beginning power computations did not include CHP production, but during the seventies the increased use of co-production implied that CHP production to some extent was incorporated in the models, too. As the technology of heat storage is relatively new it is seldom simulated by models for large CHP systems with many production units and load centers. On the other hand models do exist for local CHP schemes including a few plants, a heat storage facility, and a single load.

1.2. Formulation of tasks for this study

This project aims at the solution of one of the above mentioned problems, namely that of long-range planning of the Danish CHP production system. More precisely, the intention is to construct a tool (a computer program) that can be used in the evaluation of development plans for this system. Day-to-day heat storage in connection with CHP production, power production from wind turbines, and power connections to neighbouring countries should be included. This computer program is called Simulachron.

The Simulachron model is intended to supplement more overall models based on load-duration curves. It should therefore be so detailed that many of the strategies and methods used in the daily operation of the system could be reflected in the simulations of the operation of the future CHP production.

By means of such a tool many problems concerning the co-production of heat and power can be considered. In the normal operation of CHP plants heat production follows the load. This implies that there always is a minimum power below which electricity production cannot take place (the greater the heat demand, the greater the minimum electric power production). Therefore, a problem may arise at certain periods (e.g. late at night during a cold winter) where the demand for heat far exceeds the demand for electricity.

Another aspect of co-production specific to extraction power plants that Simulachron must be able to handle, is the dependence of the maximum power production on the actual heat produced (the greater the heat production, the smaller the maximum power production). As peak heat and peak power demands often occur at the same time of day (in spring and autumn) this is a (minor) drawback which must be taken into account when planning the expansion of the power production system*).

By comparing the simulations of scenarios with and without a certain plant or technology (heat storage, wind power, nuclear power) the benefit of this plant or technology can be examined more thoroughly than can be done by simpler existing models. The more detailed description will give a continuous record of fuel consumption and operating costs both for the individual plants and for the full system.

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*) Because of this reduction of the maximum power production, an extraction power plant is not "worth" its rated power capacity when the security of power supply is assessed for the entire system.

One of the above-mentioned simpler models is called DES*), a model of the whole Danish energy system (ref. 21). It contains a simulation part for the CHP production system, but the simulations performed are rather simple since they are based on load-duration curves and therefore cannot give the actual number of times that the various plants are started. The start-up costs are given only by a fixed fuel consumption per year for each plant. Moreover, they are based on some assumptions regarding the maximum level and time of production for the individual plants. Besides, the power-producing plants are put on load according to a fixed priority list.

A main purpose of developing the Simulachron model is to construct a tool that within reasonable limits utilizes models and methods that are as exact as possible. Therefore, it should be possible to use it in an evaluation of the assumptions on which simpler models are based and also to perform parallel simulations to find how accurate these models are.

In Simulachron the CHP plants are represented by a detailed plant model. For instance, the fuel consumption for normal operation is given as a function of the heat and power production, and the start-up fuel consumption is specified as a function of the length of the period with no production. The simulation method used should fit this plant model, that is the methods of solution used in the simulations and the mathematical description of the physical components should be consistent.

As this project does not deal with the expansion planning of the power transmission system, a rather simple representation of this is satisfactory. Only total power losses and their dependence on the production of each power plant are specified. Power flows in the individual transmission lines are not considered.

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*) DES = Danish Energy System.

To meet the selected goals it is unnecessary to investigate those fast transients of the system that are important only to the stability of the production system and to the frequency and the magnitude of the voltages. For that reason a mainly static modelling is performed allowing for changes between various (static) states once in every time step. This time step must be long enough to allow dynamic transients to die out within it - and so short that essential information on the operation of the system does not disappear. Therefore, a time step of length one to two hours will be suitable. This allows for the representation of the cooling of the boiler after a plant is taken out of operation, the time constant of the cooling process ranging from a few hours to one day. On the other hand, this time step is long enough for the power plants to be brought from minimum to maximum production. This follows from the ability of the output of a power plant to be changed normally several percent per minute. If the time step gets much smaller than one hour, the maximum rate of change of the power production must be considered. Therefore the model is prepared for a limitation on the rate of change of the production. The formal representation of the system and the simulation method will then allow for a somewhat smaller time step.

With time steps on the order of one to two hours the fast fluctuations in the wind-generated power from a single wind turbine cannot be represented. But on the other hand, the spatial distribution of windmills will cause a smoothing of the statistical fluctuations, so that the total output from many windmills distributed over a wide area will show a rather stable behaviour (refs.17, 18, and 20).

During the optimization of the unit commitment and load dispatching it is sufficient to represent the power demand by a deterministic time series. Likewise no stochastic outages of the production units are simulated. These outages are taken into account by requiring a certain spinning reserve, which also could be used to take up unforseen load increases. In a revised version of the Simulachron model it would be a rather simple task to include random variations in energy demand.

Due to the detailed modelling Simulachron can also be used in the fuel purchase planning to give an estimate of the amount and type of fuel needed at each power plant site. Moreover it can be utilized in the maintenance scheduling to find the production costs associated with various maintenance schemes.

This section is concluded by briefly enumerating the tasks to be completed:

1. For the previously given purposes (mainly evaluation of development plans) a detailed model of the CHP production system should be developed (including heat storage facilities, wind power generators, and power connections to the neighbouring countries). The transmission systems for heat and power could be modelled rather simply, and the demands for heat and power should be given as deterministic time series.
2. From this model a computer program (called Simulachron) should be developed to simulate the CHP production.
3. To check the results from Simulachron the behaviour of the existing CHP production system should be simulated.
4. Simulachron should, if possible, be compared with other programs (in the development of which there has not been put equal emphasis on CHP production or heat storage, or in which other simplifying assumptions have been made).

1.3. To which extent have the tasks formulated been accomplished?

A detailed description of the CHP system is developed in Chapter 2, and in Chapters 3, 4, and 5 a model for the operation of the system is set up. This model - Simulachron - has been programmed in FORTRAN-66.

In Section 6.2 some small and illustrative test runs of Simulachron are documented, and in Section 6.3 the operation of the total ELSAM CHP system is simulated. This system comprises some 30 power plants, including 17 CHP plants serving 7 district heating areas, and two day-to-day heat storage facilities. The operation of the ELSAM system is simulated for two periods of one week each, representing the operation during the summer and winter.

The differences and similarities between Simulachron and a model called SIM (ref. 23) used by the ELSAM power company are pointed out in Section 6.3.4. A comparison with results obtained from runs of SIM is carried out and it is shown that the differences in the results can easily be explained by the different ways in which the operation of the system is modelled.

Models for wind-generated power and for power lines to neighbouring countries have not been set up but could be included in a future extension of the Simulachron model (see below).

1.4. Further work

As noted above, wind-generated power and power transmission lines to neighbouring countries are not yet covered by the Simulachron model.

Wind generators could best be simulated by using measured time series for wind speeds at different sites to synthesize a time series for the total wind-generated power. To establish the power which has to be generated by the conventional production system the demand is simply reduced by the wind-generated power. Power lines to neighbouring countries could be modelled in different ways depending on the actual contract between the two power companies exchanging power. Some contracts could be modelled by changing the load curve (partly levelling it out) before the simulation of the power production starts, while other contracts would be modelled as a power plant with negative minimum "production" and perhaps a time-dependent specific "production" cost.

From a power production point of view Denmark is today divided into two separate systems by the Great Belt with no interactions. In the future these systems will probably be connected by one or more high voltage DC cables. In a simulation of the total Danish power system the rated transmission capacity of the cables should be considered.

Considering CHP plants a model for gas turbine CHP plants could be introduced. Also the ability for extraction and back-pressure power plants to by-pass the turbine and feed the steam directly to the heat exchanger could be modelled.

In the present version of Simulachron the power demand is given as a deterministic time series. However, for the optimum production schedule thus found it would be possible to introduce a stochastic load as a pertubation of the deterministic load. Moreover, it would be possible to introduce the reliability of the plants. Thus, when the stochastic nature of the load and the reliability of the plants are utilized the expected time with insufficient production capacity (specified by LOLP = loss-of-load probability) and the expected amount of unserved demand for electric energy could be found.

2. MATHEMATICAL REPRESENTATION OF THE COMPONENTS OF THE HEAT AND POWER PRODUCTION SYSTEM

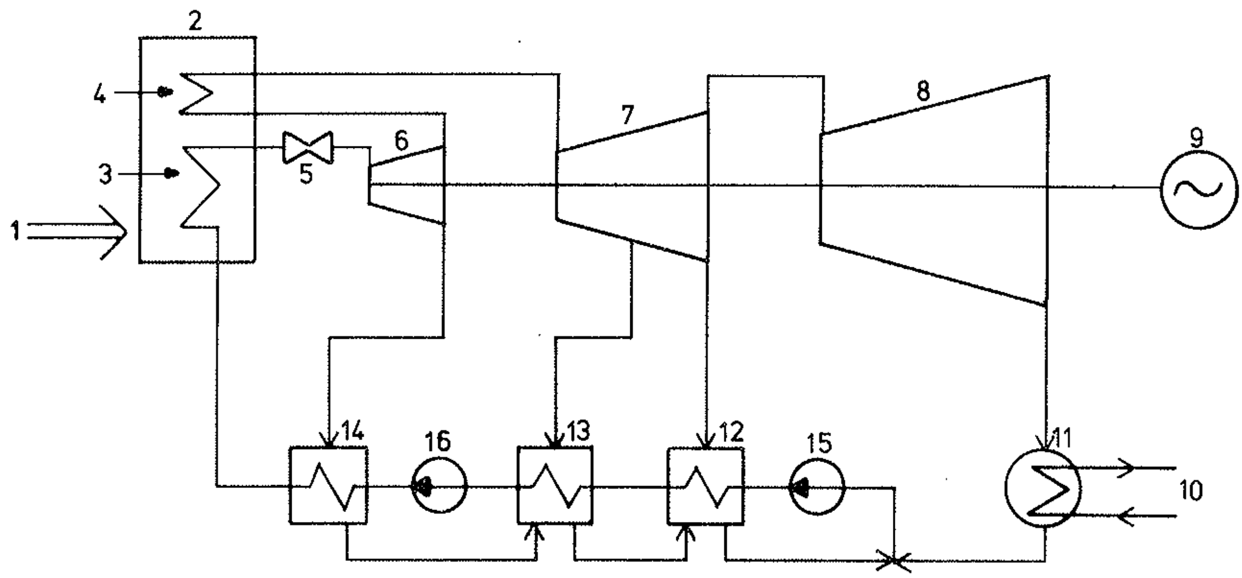
2.1. Introduction

In this chapter a description will be given of the representation of the various technical components in the model. This comprises power plants, co-generation plants, boilers, heat storage facilities, and transmission systems for heat and electricity. The representation is not meant to be used in a dynamic simulation of the power system but rather in a static simulation where plants are assumed to produce at constant levels throughout each time step of one-to-two hours. The transitions between these constant levels are not considered but are assumed to be feasible from a technical point of view. This assumption is valid for dynamic transitions with time constants smaller than the time step. The only time constants that have to be considered explicitly are those for the cooling of the boilers. This problem is dealt with as described for the start-up fuel consumption in Section 2.2.

The formal representation of the production units is independent of the type of fuel used, but naturally the fuel consumption varies from unit to unit. For instance, nuclear base-load power plants and oil-fired peak-load power plants are represented in the same way, but with different fuel consumptions per produced kWh electricity and different start-up costs.

2.2. Condensing power plant units

Until the recent interest in co-production of heat and power, condensing plants have by far been the most widely used power plants. The economically feasible rated capacity of new plants varies from about 200 MW for fossil fired peak-load plants to 1300 MW for nuclear base-load plants. The thermal efficiency lies in the interval from 30% to 40%, the smaller figure being valid for peak-load plants, old plants, and nuclear plants while 40% or even more can be reached only by modern large fossil-fuelled base-load power plants. The difference in efficiency is a result of different live steam qualities (temperature and pressure) and different degrees of sophistication (reheating of steam, number of feed water preheaters etc.). Figure 2.1 shows schematically the components of a fossil-fuelled plant.



1. fuel and air	8. low-pressure turbine
2. boiler	9. generator
3. water/steam tubes	10. cooling water
4. reheater	11. condenser
5. throttle	12-14. preheaters
6. high-pressure turbine	15. condensate pump
7. medium-pressure turbine	16. feed water pump

Fig. 2.1. Simplified diagram of a fossil-fired condensing power plant.

For instance, by regulating the throttle at the inlet to the high-pressure turbine and at the same time changing the fuel input to maintain the live steam quality the generated power can be varied within certain limits, the minimum production being 20-30% of the rated power. When leaving the high-pressure turbine of base-load plants, the steam is normally reheated to the live steam temperature before being sent to the medium- and low-pressure turbines. When the steam has condensed in the condenser, it is heated in several preheaters before being pumped back to the boiler. The steam for the preheaters is extracted from the turbines. This preheating in several stages improves the overall efficiency, and thus increases the power output of the plant. The reason for this lies in the thermodynamics of the steam cycle; it can be stated by the following rule of thumb: "When heating something, don't take the energy from a heat source that is unnecessarily warm!" - otherwise the entropy will increase too much because of the temperature difference between the two heat-exchanging media.

When the steady-state power production is varied within its limits, the fuel consumption changes accordingly, but power production and fuel consumption are not exactly proportional to each other. I the upper part of Fig. 2.2 two examples of fuel consumption curves are shown. The lower part of the figure gives the corresponding specific and incremental fuel consumption curves*). The specific fuel consumption has a minimum at the layout level of production at which the whole plant is optimized. When the power is reduced from this point the amount and thermodynamic properties of the steam inlet to the high-pressure turbine is changed. Therefore, the optimal conditions are no

*) If the dependence of the fuel consumption on the power production P is given by the function F(P), the specific and incremental fuel consumption can be expressed as F(P)/P and dF(P)/dP, respectively.

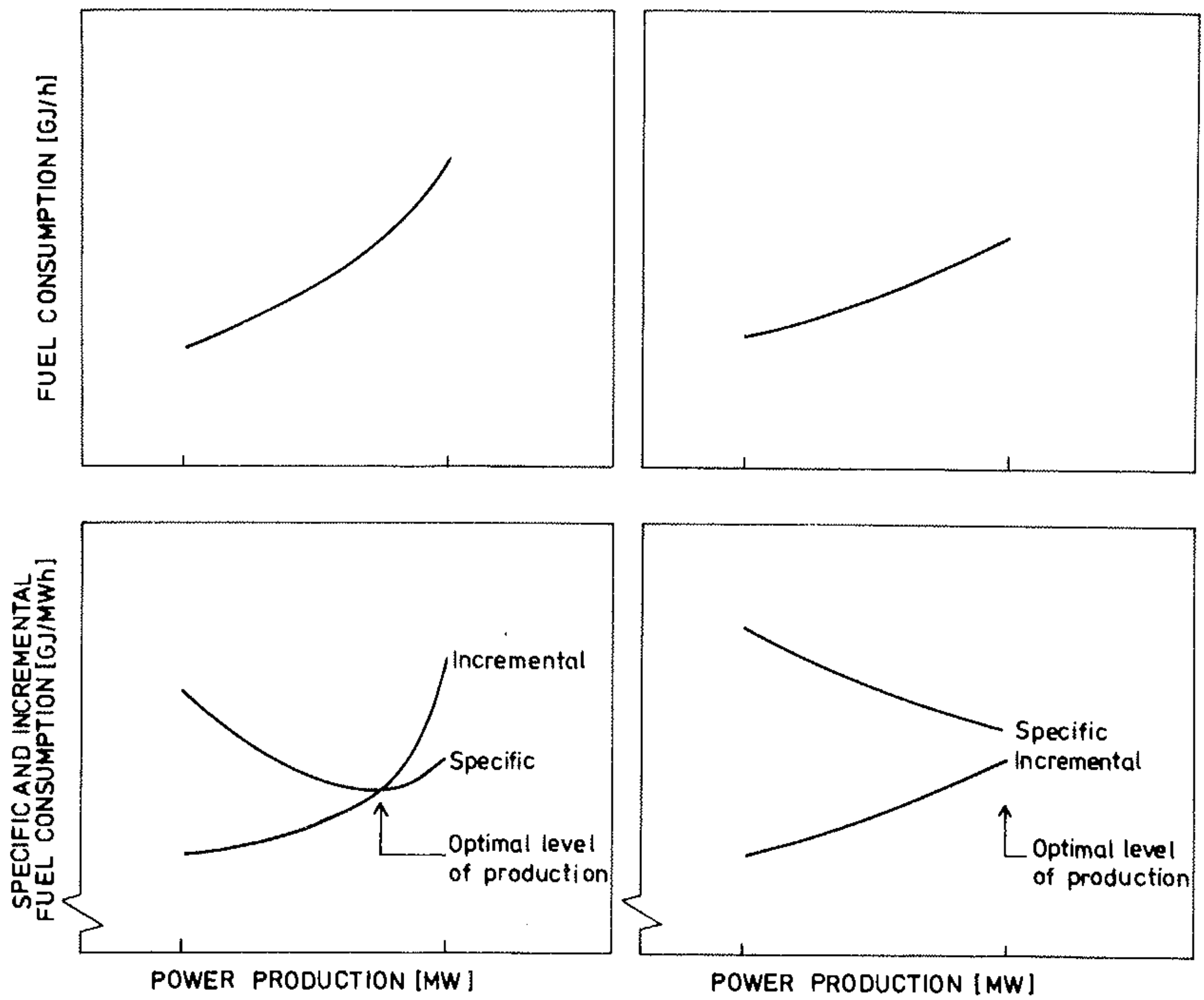


Fig. 2.2. Two examples of fuel consumption. The non-linearities are exaggerated for clarity.

longer fulfilled, and a slightly increased specific fuel consumption results. Moreover, the heat loss from the boiler particularly is independent of the level of production, and consequently it has a greater influence on the thermal efficiency at low production levels.

In the model the fuel consumption has to be given in some systematic way. Three different methods of representation can be obtained by selecting one of the following descriptions in addition to a fixed fuel consumption at minimum load:

1. A constant incremental fuel consumption from minimum to maximum load.
2. A piecewise constant incremental fuel consumption.
3. A piecewise linear, continuous incremental fuel consumption.

Incremental fuel consumption curves corresponding to the three methods are shown in Fig. 2.3.

The first method gives a very coarse description of the power plants, and if used in a load dispatching procedure it would allow only one plant at a time (the marginal plant) to produce at part load while other plants would be operating at minimum or maximum load - if operating at all.

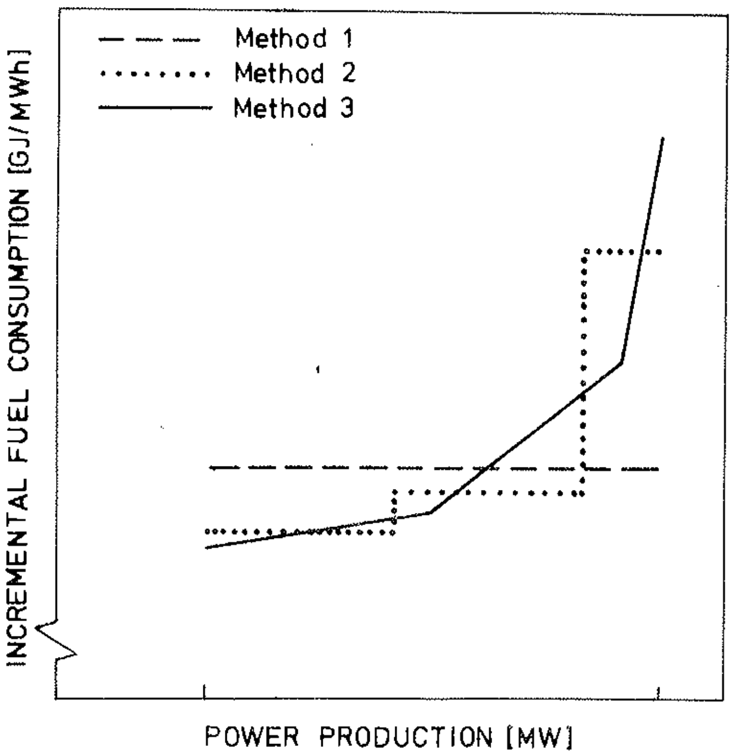


Fig. 2.3. Incremental fuel consumption.

In the same way the second method would allow the generators to produce only at certain fixed levels (at discontinuities in the incremental cost curve), while the third method can bring about any production level as a result of a load dispatching. Therefore this method is used in the model.

One drawback of the third method is that the cost curve is nonlinear (piecewise 2nd order curve) and therefore the mixed integer programming problem of the unit commitment cannot be solved by utilizing simplex-like methods well known from linear programming.

The fuel consumption could also be approximated by a polynomial of order n in the produced power. In this case the incremental fuel consumption would be given by a polynomial of order n-1. This approach could be incorporated in the Simulachron model. If n equals 3 the load dispatching could be carried out easily by solving second-order equations, but if n is greater than 3 there is no analytic solution.

The method used for load dispatching, as described in Chapter 4, requires that the incremental fuel consumption curve increase monotonically. This is fulfilled by nearly all the units in the Danish power system.

To keep the operation of the boiler stable some coal-fired plants have to be fuelled by oil (partly or totally) at production levels near their technical minimum as shown in Fig. 2.4.

When the incremental fuel consumption curves for these plants are changed to incremental cost curves by multiplying them by the correct fuel prices, one gets not only a descending incremental cost curve, but also a descending total fuel cost curve. This could be modelled by introducing integer variables specifying in which interval (oil, oil/coal, coal) production takes place. However, to reduce the complexity of the model it has been chosen to change the description for these plants by ignoring the oil consumption, or by levelling out the abrupt change in costs.

When a power plant has not been operated for some time it takes some extra fuel to get it into operation again. This fuel consumption depends on the length of the period without production, mostly because of the cooling of the boiler. Therefore a "cold start-up" fuel consumption is specified together with the time constant of the cooling process. Thus, the fuel needed for a start-up can be found as shown in Fig. 2.5.

The costs for running a power plant comprise fuel, operation and maintenance (O&M). Therefore also variable (labour hour per hour of production and per MWh produced) and fixed O&M costs (labour hour per kW rated capacity per year) must be given together with fixed start-up costs (labour hour per start-up). Investments are not considered in the model.

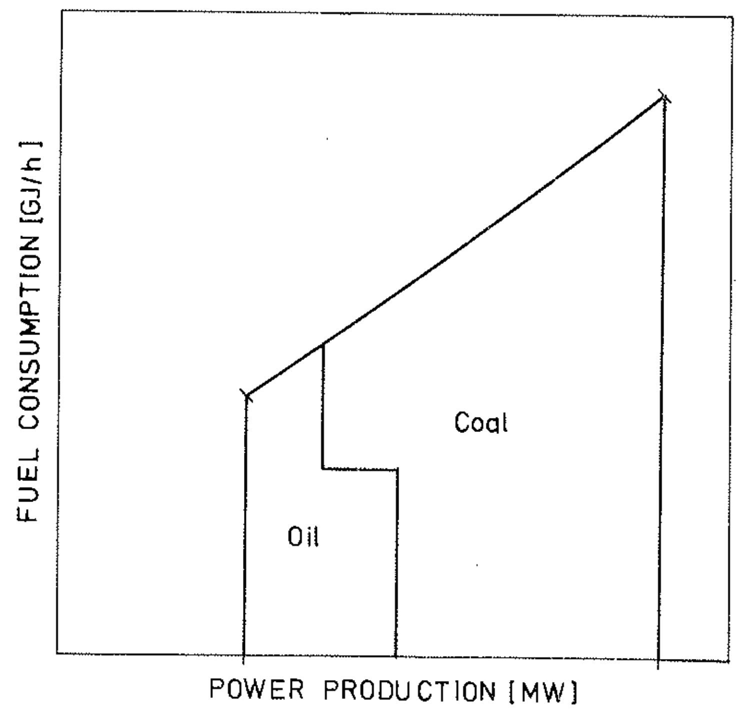


Fig. 2.4. Oil consumption at coal-fired power plant.

Variable and fixed O&M costs, fixed start-up costs and prices of the various fuels and of labour are given for a base year together with separate inflation factors.

Periods of planned outages can be given for any unit while unplanned outages are taken into account by specifying reliability factors for the units. In the model forced outages are not simulated but the reliabilities can optionally be introduced in the unit commitment by demanding that not only the sum of the rated powers, but the sum of the rated powers multiplied by the reliability for the running generators must at least match load and losses.

The load following capability of the plants (the rate at which the power production can be changed) is not considered in the present model. This follows because the production of most plants can be changed by several percentage points of the rated capacity per minute, that is from minimum to maximum production within the time step used. Nuclear power plants are unable to change their production that fast, but because of their relatively small incremental fuel costs the optimization of the operation of the whole system will commit them to produce at an almost constant level near their rated capacity.

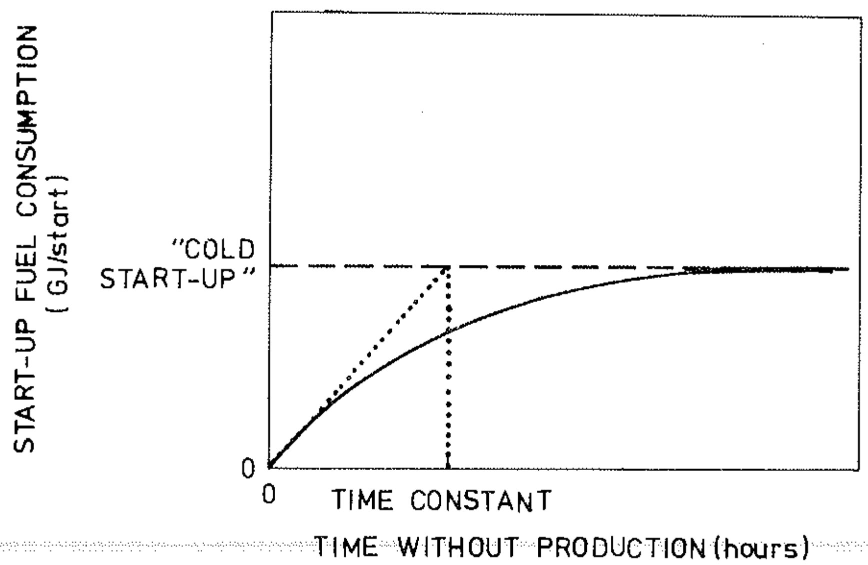


Fig. 2.5. Start-up fuel consumption.

2.3. Back-pressure power plant units

Due to the inevitable transformation of exergy into anergy in the boiler of a conventional condensing power plant 60-70% of the energy in the fuel is lost in the conversion of the fuel energy into electricity. Most of the lost energy is transferred to the cooling water in the condenser at a temperature too low for any practical use (except fish farming and greenhouse heating perhaps).

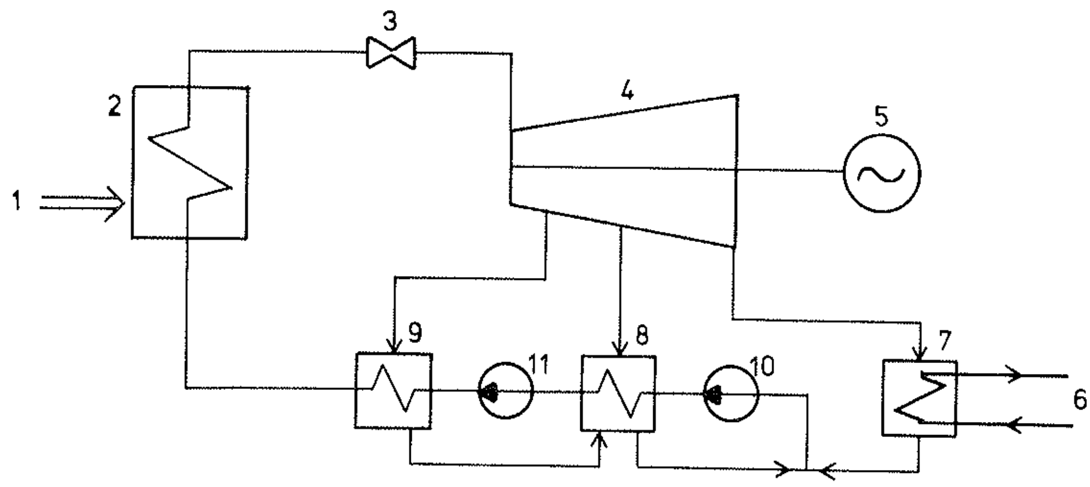
In principle, what is done in a back-pressure power plants is to raise the temperature in the condenser to a level high enough to use the disposed heat in, e.g. a district heating system or some industrial process. The increased temperature and pressure at the outlet of the low-pressure turbine brings about a reduction of the electricity produced, but the total efficiency of 30-40% for condensing power plants is increased to 80-85% for back-pressure power plants.

The components of a back-pressure power plant are shown schematically in Fig. 2.6.

The figure shows a back-pressure power plant with only one heat exchanger for the district heating water. For larger plants (e.g. 100 MW) two heat exchangers might be economical, the steam for the second heat exchanger being extracted from the turbine. To maximize the power output the temperature increase should be divided equally among the heat exchangers.

The rated capacity of back-pressure power plants is normally less than 100 MW, and therefore the construction is often simpler than that of most condensing power plants: The live steam has a lower pressure and temperature, there is no re-heater, and there is a smaller number of preheaters.

For a constant forward temperature of the district heating water the power production at a back-pressure plant is very close to being proportional to the heat production. In the model this



1. fuel and air
2. boiler
3. throttle
4. turbine
5. generator
6. district heating water
7. heat exchanger
8-9. preheaters
10. condensate pump
11. feed water pump

Fig. 2.6. Simplified diagram of a back-pressure power plant.

proportionality is assumed to be perfect (Fig. 2.7). The ratio of power production to heat production will hereafter be called "the back-pressure constant", designated by c_m.

For back-pressure power plants with more than one heat exchanger it is possible to raise the temperature of the district heating water to a certain range of temperatures. Each value of the forward temperature has a corresponding value of the back-pressure constant. The higher the temperature, the smaller the back-pressure constant.

It might also be possible to feed live steam directly into the heat exchanger without any power production. None of the two options mentioned (varying forward temperature, and live steam directly to the heat exchanger) is implemented in the model.

The back-pressure power plant can be compared with a fictive condensing power plant, that is equivalent to the back-pressure

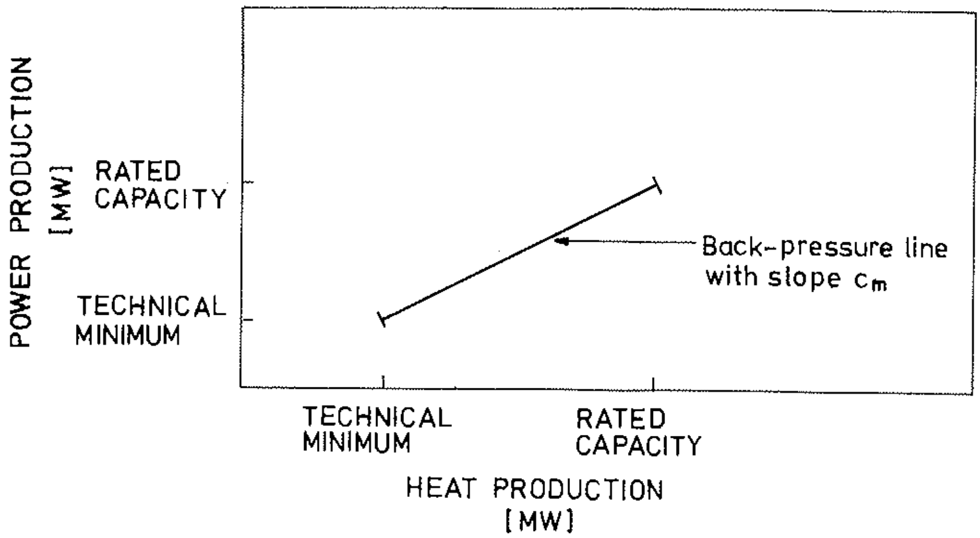


Fig. 2.7. Power diagram for back-pressure power plant.

plant except that the steam, instead of being led to a heat exchanger for district heating purposes, is allowed to expand further in a low-pressure turbine before being condensed in a normal condenser.

By introducing the condensation temperature and thermodynamic efficiency of the equivalent condensing power plant, the back-pressure constant can be expressed in the following formula (ref. 10):

$$c_m = \eta_{gen} \cdot \eta_{mech} \cdot \{1/[(T_{back}/T_{con})\cdot(1-\eta_{con})]-1\}$$

Variables related to the back-pressure plant:

- c_m : back-pressure constant (0.3-0.8).
- η_{gen} : efficiency of generator (e.g. 0.98, includes electrical and mechanical losses in the generator).
- η_{mech} : efficiency of turbine (e.g. 0.99, includes mechanical losses in the turbine).
- T_{back} : thermodynamic mean temperature (K) of the condensate in the heat exchanger for the district heating water.

Variables related to the equivalent condensing plant:

η_{con} : thermodynamic efficiency (0.3-0.4, excludes mechanical and electrical losses).

T_{con} : condensation temperature (K) (e.g. 30°C=293 K).

The temperature T_{back} is given by (ref. 10):

$$T_{back} = T_F + \Delta T - (T_F - T_R) \cdot (n-1)/(2 \cdot n)$$

T_F = forward temperature of the district heating water (e.g. 90°C).

T_R = return temperature of the district heating water (e.g. 40°C).

ΔT = temperature difference between the condensated steam and the district heating water leaving the heat exchanger (e.g. 5°C).

n = number of heat exchangers used to heat the district heating water (n equals one or two, and occasionally three).

These formulas for c_m and T_{back} are invalid if the steam for the district heat exchangers is superheated.

Figure 2.8 (ref. 9) shows how c_m depends on the forward temperature and on the temperature and pressure of the live steam.

It can be found from the figure that the back-pressure constant, and thus the power production, is augmented by 5-6% by introducing a second heat exchanger. The figure also shows that c_m is reduced by approx. 0.027 if the dimensioning forward temperature is increased by 10°C.

This section deals with back-pressure power plants, and in the preceding part of the section various characteristics have been considered that separate these plants from condensing power plants. But concerning fuel consumption, start-up costs, and

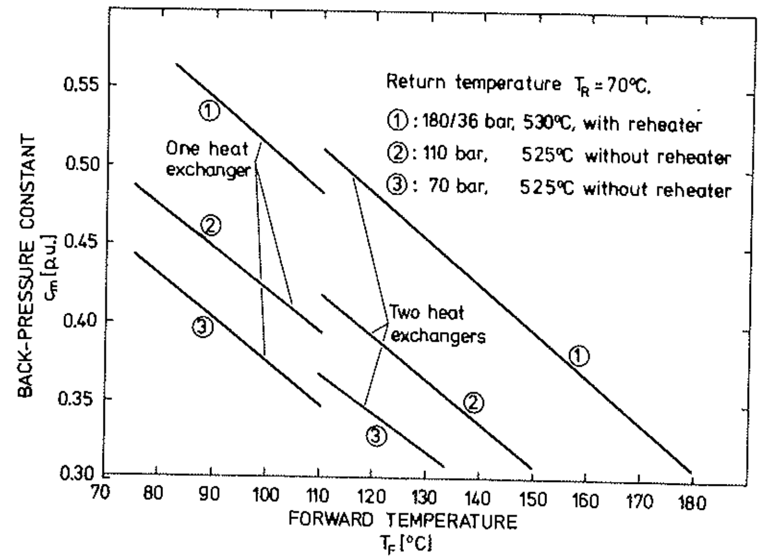


Fig. 2.8. Back-pressure constant for back-pressure power plants. Steam flow 70-280 kg/s.

operation and maintenance costs the specifications in the model are like those given for condensing plants.

The relative losses in the heat transmission system from plant to consumer are specified separately for each co-generation plant.

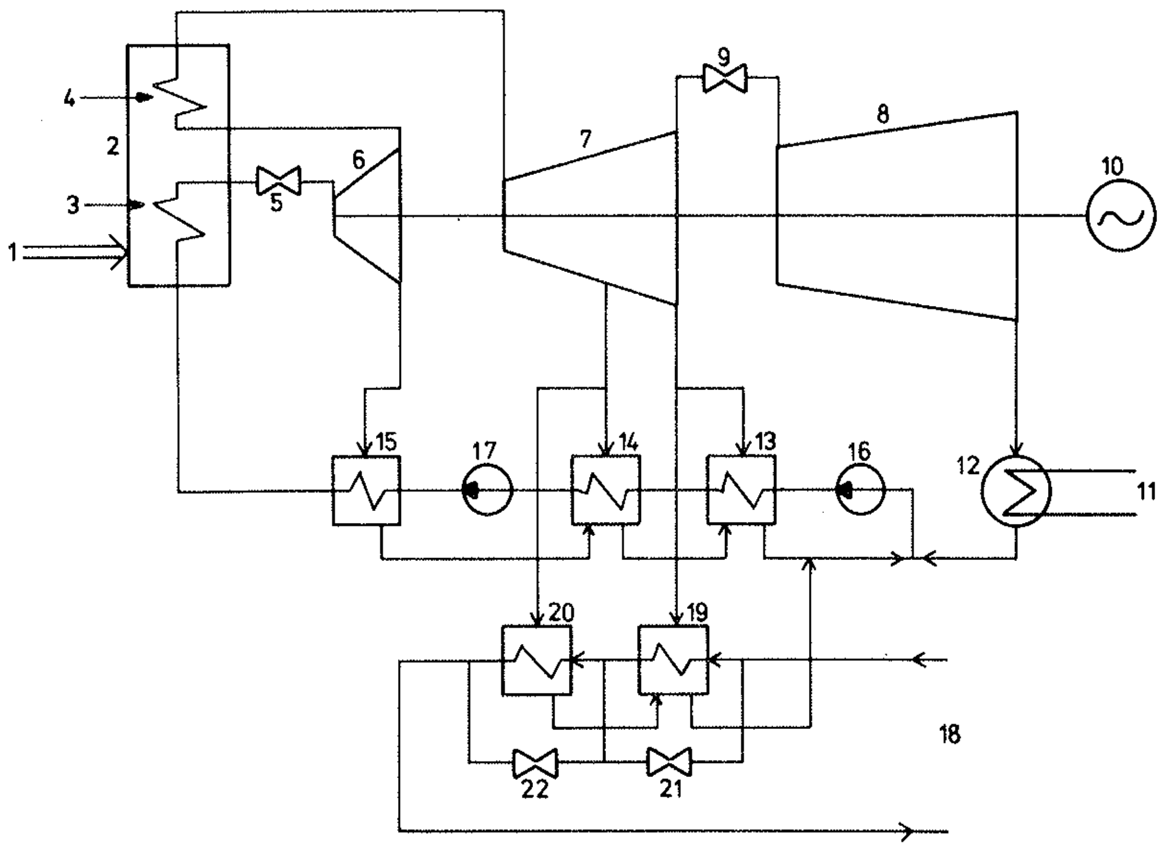
To lower the labour costs it might be economical to shut down small back-pressure power plants late at night when the demand is reduced. This possibility is implemented in the model.

2.4. Extraction power plant unit

An extraction power plant can be considered as a combination of a condensing power plant and a back-pressure power plant, but its operational flexibility is much greater than that of a back-pressure power plant: It is possible within certain limits to specify the heat and power productions independently.

Figure 2.9 gives an example of how an extraction power plant might be constructed.

Many extraction plants are former condensing plants that have been changed into extraction plants by retro-fitting a heat exchanger. The steam is then taken from the outlet of the medium-pressure turbine. Such plants can have one heat exchanger only, since the extraction arrangements of the medium-pressure turbine are originally dimensioned only for preheating the feed water, and not for district heating purposes.



1. fuel and air
2. boiler
3. water/steam tubes
4. reheater
5. throttle
6. high-pressure turbine
7. medium-pressure turbine
8. low-pressure turbine
9. throttle

10. generator
11. cooling water
12. condenser
13-15. preheaters
16. condensate pump
17. feed water pump
18. district heating water
19-20. heat exchangers
21-22. bypass throttles.

Fig. 2.9. Simplified diagram of an extraction power plant.

In the model fuel consumption, start-up costs, operation and maintenance costs, and relative losses in the heat transmission system are specified in the same way as for back-pressure power plants.

When operating an extraction power plant the heat and power outputs can be specified independently within certain limits as shown in Fig. 2.10 (from ref. 12). Both the correct nonlinear limitations and the linear limitations used in the model are given.

The technical reasons for the limitations (with indices defined in Fig. 2.10) are stated briefly below:

1. In contrast to what is the case for back-pressure power plants, the heat production can be reduced to zero by letting all steam (except that needed for the preheaters) go on expanding in the low-pressure turbine. Thus, it is operated as a condensing power plant. This can be accomplished either by stopping the district heating water to the heat exchangers or by closing the valves (not shown in Fig. 2.9) in the steam lines to the heat exchangers.
2. The maximum heat power is given by the dimensions of the heat exchangers and the steam-extraction arrangement.

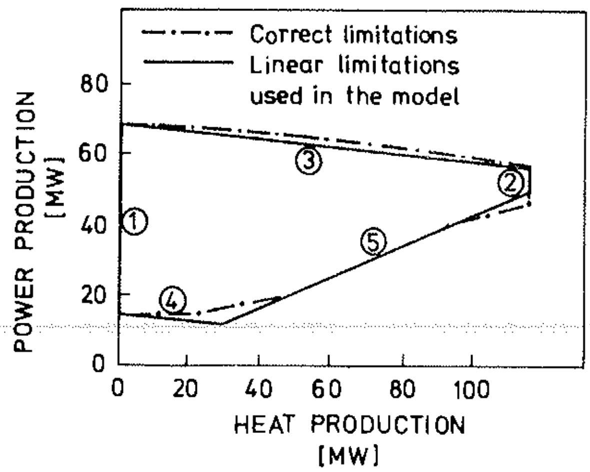


Fig. 2.10. Example of power diagram for extraction power plant.

3. This upper limit for the power production corresponds to full load operation of the boiler and high-pressure turbine. It is seen that the maximum power production decreases as the heat production is increased.
4. This lower limit for the power production is given by the minimum load at which the boiler can be operated.
5. This limit is reached when all steam is extracted to the heat exchangers, except what is needed to cool the low-pressure turbine (approx. 5% of the steam). Operation on this limit, "the back-pressure line", is equivalent to the operation of a back-pressure power plant. The cooling of the low-pressure turbine is necessary to remove the frictional heat generated in the turbine.

Figure 2.11 shows how the nonlinear curves for constant fuel consumption are linearized in the model in the same way as the limitations of operation.

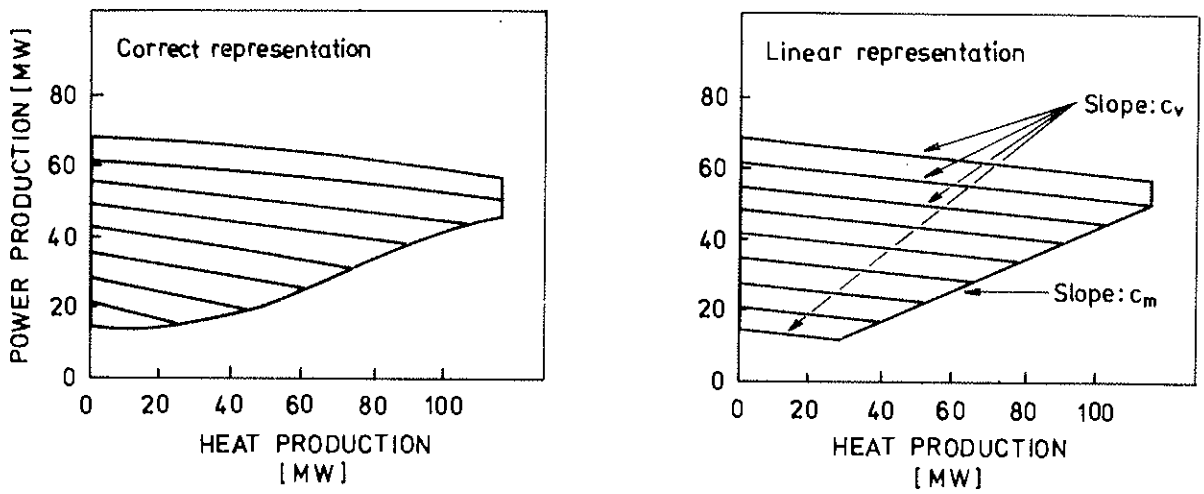


Fig. 2.11. Example of power diagram with equi-fuel-consumption curves for extraction power plant.

As for back-pressure power plants the slope of the line corresponding to back-pressure operation is called the "back-pressure constant", c_m. The slope of the parallel equi-fuel-consumption lines is designated by c_v. This constant also tells how much the maximum power production is reduced when the heat production is increased. The following equation gives an expression for c_v (ref. 11):

$$c_v = \eta_e/\eta_t - (1 - \eta_e/\eta_t) \cdot c_m$$

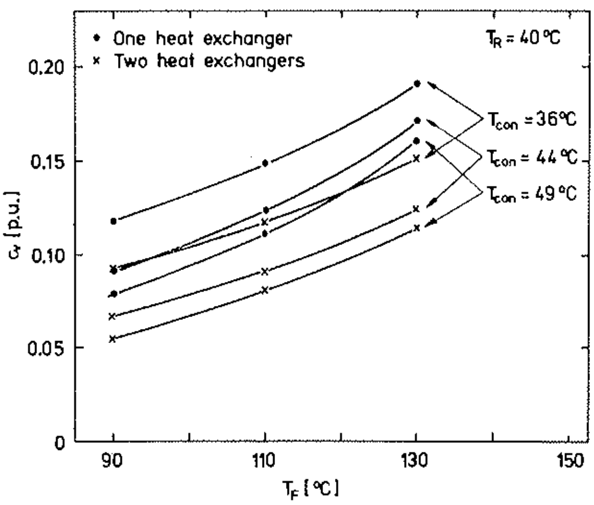
where

c_m: back-pressure constant

η_e: efficiency for power production at zero heat production (η_e=0.3-0.4)

η_t: total efficiency for production at the back pressure line (η_t=0.8-0.9)

Figure 2.12 gives c_v for a fossil-fuelled extraction power plant with rated power 300 MW and equipped with a reheater. (Live steam:



T_F : forward temperature of the district heating water.
T_R : return temperature of the district heating water.
T_{con}: thermodynamic mean temperature of steam in condenser.

Fig. 2.12. Example of the c_v-constant for extraction power plant.

180 bar, 530°C. After reheater: 36.5 bar, 530°C). The figure which is a reproduction of tables in ref. 9, gives c_v in the case of one and two heat exchangers, and as a function of the ther-

modynamic mean temperature T_{con} of the steam in the condenser and of the forward temperature T_F of the district heating water. The curve for $T_{con} = 36^oC$ is valid for Danish power plants cooled by sea water during the summer, while the two other curves correspond to plants equipped with cooling towers, structures that are not used in the Danish power system. It is seen that introducing two heat exchangers instead of one will decrease c_v, as will a reduction of the forward temperature.

2.5. Diesel engines and gas turbines

The formal description of condensing power plants used in the model is so general that it can be used without alteration for generators driven by diesel engines or gas turbines as well, but of course, the actual data are different. For instance, any shape of the incremental fuel consumption curve can be approximated to any degree of accuracy by a piecewise linear curve by increasing the number of linear segments.

Waste heat from diesel engines and gas turbines can be utilized by exhaust coolers. For diesel engines heat from oil coolers can be extracted for district heating, too. This co-production of heat and power can to a certain extent be modelled in the same way as CHP production at extraction power plants.

2.6. District heating boilers

Compared with power plants, district heating boilers are represented in a rather simple way in the model. It is assumed that the efficiency of a boiler is constant over the range of operation, and it is therefore given as only one value. Likewise, the rated heat power and the relative losses in the transmission of heat to the consumers are given.

Variable and fixed operating costs are specified for a base year together with yearly inflation factors.

Refuse incineration plants are treated as district heating boilers with (very cheap or perhaps negative-cost) "refuse" as fuel.

It is possible to specify the time of the day when the unit is available. This makes it possible to simulate a shut-down of a refuse incineration plant during the night.

2.7. Heat storage facilities

In the model only day-to-day and not seasonal storage facilities are considered. In the Danish energy system there are so far only two heat storages, both of which are constructed as insulated water tanks with hot water in the top and cold water at the bottom.

In the model they are represented in the following way: The capacity of each storage is given as the maximum energy that the storage can hold. Also given is the maximum heat power that can be put into or taken out of the storage. These two maximum heat powers (into and out of the storage) do not have to be equal. They depend on the pumping capacity and on the requirement that the stratification of the water in the tank should be undisturbed.

The heat losses from a storage is specified as the sum of two terms: a fixed loss independent of the heat content, and a term proportional to the content. The relative losses in the transmission system from the storage to the consumers are given, too.

The operating costs for the heat storage (kr per MWh into or out of the storage) are given for a base year together with an annual inflation factor. When specified in this way the operating costs can include the pumping power.

2.8. Transmission system for electricity

The transmission network can be represented by a set of conductances and susceptances. The associated equations that link together the voltages and powers (active and reactive powers) are nonlinear (quadratic) and can therefore be solved only by iterative methods.

The use of such a detailed representation in an optimal load flow would be too time consuming in the computer simulations, because the simultaneous unit commitment and load dispatching call for a very large number of load dispatchings. Therefore, a more easily handled approximative representation is used. This representation, called the B-constant method, gives only the total loss in the transmission system. Therefore, the powers through individual lines and the voltages at the busses are not found.

The losses are given by a second-order expression:

$$P_L = \sum_{i=1}^{NBUS} \sum_{j=1}^{NBUS} P_{Bi} B_{ij} P_{Bj} + \sum_{i=1}^{NBUS} B_{oi} P_{Bi} + B_{oo}$$

where P_L : total losses (MW).

P_{Bi} : active power from generators connected to bus no. i (MW).

NBUS : number of generator busses.

B_{ij} : second-order B-constants (MW^{-1}).

B_{oi} : first-order B-constants (MW/MW).

B_{oo} : zero-order B-constant (MW).

The incremental losses that are to be used in the load dispatching procedure, can readily be found from the above equation:

$$\partial P_L / \partial P_{Bi} = 2 \cdot \sum_{j=1}^{NBUS} B_{ij} P_{Bj} + B_{oi}$$

From different approximations to the exact load flow equations various sets of B-constants can be calculated. In ref. 1 and ref. 2 several methods are discussed.

Reference 2 shows that one of the best methods is described in ref. 3. This method is as exact as others, but does not call for load flow solutions, and yet the same set of B-constants can be used for different loads. The relative error in the losses is less than 20%, which means that the absolute error is less than 1% of the load. Examples in ref. 2 show that errors in total production costs (for a system of 8 generators and 39 busses) found by using B-constants in the load dispatching are less than 0.1%.

3. MAIN CHARACTERISTICS OF THE COMPUTER MODEL

3.1 Introduction

In this chapter a short discussion of the computer model will be given. A more detailed description of the various parts of the model can be found in subsequent chapters.

The modelling of the total CHP system is very comprehensive. It takes into account the time-varying deterministic demands for heat and power, and satisfies these demands in the cheapest possible way.

A time period of arbitrary length divided into an arbitrary number of time steps can be simulated. This time period is assumed to be cyclic in the way that the first time step follows the last one. By doing so it is unnecessary to specify the situation at the beginning or at the end of the period in question.

The time steps in which the period is divided interact in such a way that the minimization of the costs cannot be performed independently for each time step. Instead, it is necessary to consider all time steps simultaneously.

The interaction of the time steps stems from two different sources. Firstly, the fuel needed to bring a power plant into operation depends on the length of the period without production (see Section 2.2), and secondly, heat storage makes it possible to move heat production from one time step to another, i.e. the production does not have to follow the demand, but of course the total energy supplied within say one week (production less losses) must match the energy demand.

When simulating the operation of a CHP system over a long time, say a whole year, each week is considered separately to keep down the number of variables that have to be optimized simultaneously. Because the heat and power load profiles change very little from week to week, and because a week, as mentioned, is considered to be cyclic, the error introduced by breaking down the year in independent weeks is small. Figure 3.1 gives an example of how the simulation of a year is broken down into time periods and time steps.

When operating a production system comprising condensing power plants and CHP plants the combined production has a priority over the production at the condensing power plants. This follows from the economic savings that are brought about by producing as much heat and electricity as possible in combined production.

In the computer model the same philosophy is followed: First the heat production (at CHP plants and district heating boilers) and thereafter the power production is simulated. In order to reach

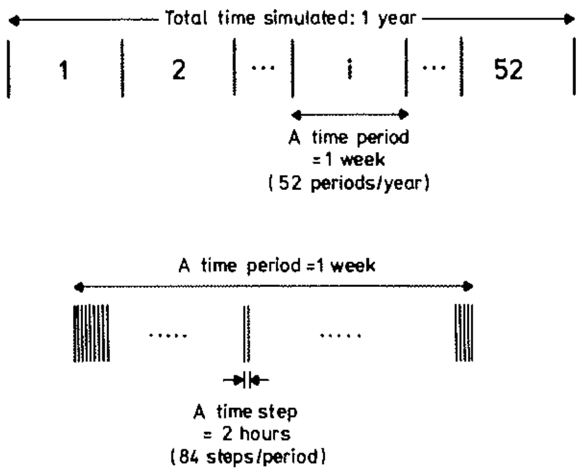


Figure 3.1. Example of "time period" and "time step": A year is divided into 52 non-interacting, cyclic periods of one week. Each period is divided into 84 steps of 2 hours. All quantities (load, production, heat, electricity) are constant within a time step.

the total optimum operation it might be necessary to iterate around these two sub-optimizations until the solution is stable.

Figure 3.2 shows the main parts of the computer model. The simulations start at the top of the figure whereafter the lower part is considered. As can be seen, electricity from wind generators can also be included in the simulation. The above mentioned iteration is not shown.

3.2. Simulation of heat production

The simulation of the heat production is performed for one district heating area at a time. In this context a district

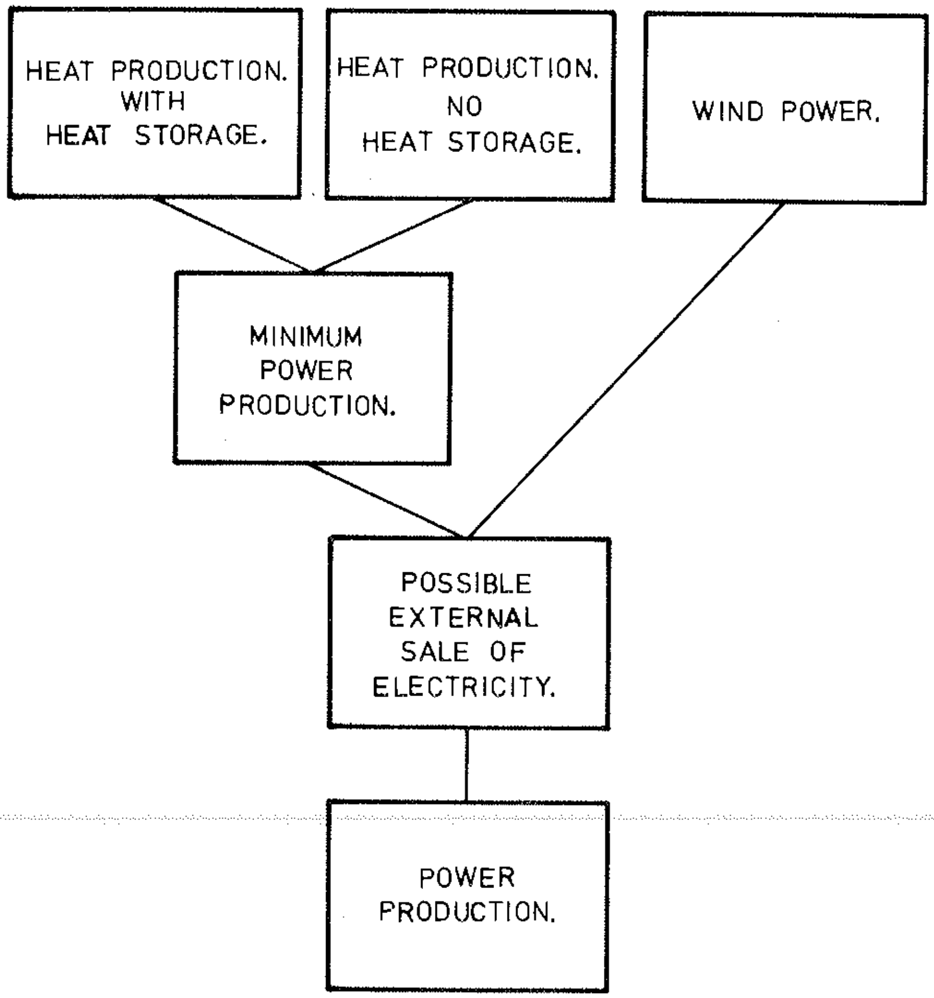


Fig. 3.2. Main parts of computer model.

heating area is defined as a system consisting of a district heating grid with a heat demand and production system that might comprise district heating boilers, back-pressure power plants, and extraction power plants. A heat storage facility for day-to-day storage of hot water is optional, too.

For each time step, the computer model dispatches the heat load among the heat-producing units. If a heat storage is present in the actual district heating area, the production can be moved from one time step to another within the limits of the storage.

To find the production strategy which brings about the minimal cost one must take into account the value of the co-produced electricity from back-pressure and extraction power plants. The price of this power is considered a given, time-dependent variable during the simulation of the heat production.

3.3. Interface between the modelling of the heat and power production

When the simulation of the heat production has been performed for all district heating areas, an overall simulation of the electricity production is carried out. During this latter simulation the heat production is left unchanged.

Therefore, the schedule for the heat production brings about a minimum power production below which the CHP units cannot be operated. For back-pressure power plants the power production is fixed when the heat production is given. In contrast to this, extraction units still have some operational flexibility when the heat production is kept constant.

The sum of these minimum power productions is found, and if greater than the power demand, the excess production is assumed to be sold to some external power grid at a time-varying price.

For situations where the excess power production is sold to a neighbouring grid at a fixed price given in a contract for the cooperation, this price should be used. In other situations where the grid in question and the neighbouring grid are operated at a common marginal cost of production, this marginal costs as found in the simulation of the power production should be used.

If wind-generated electricity is introduced in the grid the total power from the wind turbines is subtracted from the load, and thus the introduction of windmills might increase the external sale of electricity.

3.4. Simulation of power production

During this part of the computation the heat production is kept constant, as mentioned earlier. Therefore, back-pressure plants do not take part in the optimization, and extraction plants can be considered as condensing plants. Figure 3.3 shows the limits of operation for an extraction unit.

The simulation of the power production consists of a simultaneous unit commitment and load dispatching. In the optimization of the unit commitment it is found for each time step which plants are to be in operation. This gives rise to the introduction of zero-one variables specifying whether a particular generator in a given time step is operated or not. The necessity of integer variables is accountable to a minimum level of production (20-30% of their rated capacity, see Section 2.2) that power plants have below which they cannot produce. Extraction plants that are committed to produce heat are required to produce electricity as well.

In the load dispatching it is found for each time step how much power those units that are committed for operation should produce to minimize the costs.

Although described separately here the optimization of the unit commitment and load dispatching take place at the same time in the model. This is more thoroughly described in Chapter 4.

As one of the results of the load dispatching one gets a time series of marginal power costs. This time series is fed back to the heat production simulation which is then repeated. During this renewed simulation the time series is used to find the value of the co-produced electricity. In this way an iteration is performed around the heat and the power production parts of the model until the solution is stable.

When simulating the heat production for a district heating area with a heat storage facility this iteration is essential to find the optimal strategy for the operation of the storage unit. If no heat storage facility is present in any district heating area it is sufficient to perform only one or two simulations of the heat and power production. That is, one has to feed back the marginal power costs one time at the most.

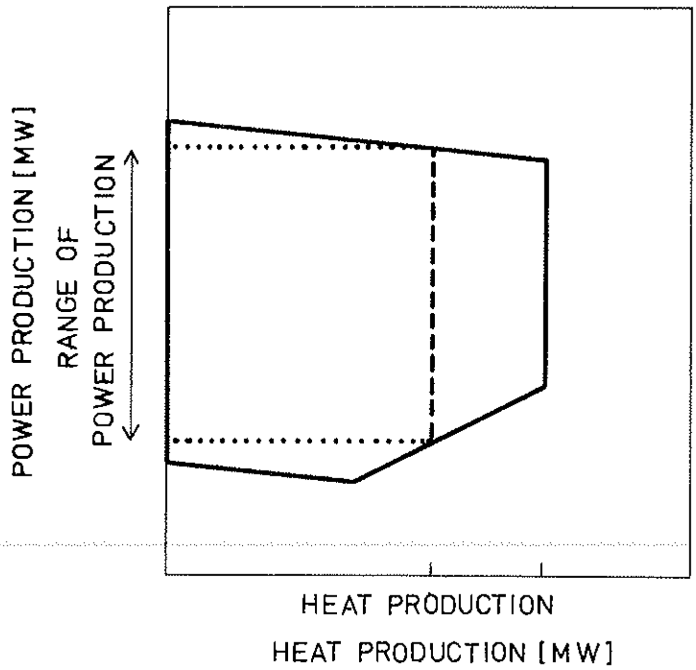


Fig. 3.3. The heat production determines the possible range of power production for an extraction power plant.

4. POWER PRODUCTION

4.1. Introduction

Since power plants cannot be operated below a certain level of production (unless stopped) one must, when planning the power production, perform a unit commitment for each hour of the day (i.e. decide which units are to be operated).

Because of the time needed to start a power plant one actually has to look ahead some hours and make an estimate of future demand. However, in the model discussed in this report the demand is assumed to be deterministic and given by a fixed time series. In a future extension of the model a stochastic load function might be incorporated.

The object of the generating unit commitment is to ensure that there are always sufficient units in service to meet the demand with adequate reserve margins. On the other hand, to minimize the running costs, not too many units should be committed to production since this would force them all to be operated at part load, a less economical operation than full load.

Each start-up of a power plant brings about extra costs, namely fixed start-up costs and additional fuel costs. When a power plant is stopped the boiler will cool unless special action is taken. Therefore the start-up fuel consumption depends on the time elapsed since the unit was stopped, as described in Chapter 2.

The above short description of the unit commitment problem shows that in the modelling of this procedure one has to introduce zero-one variables to describe whether a certain plant is operated or not. The mathematical formulation will be given in Section 4.2.

When simulating the operation of the power production system for say one week the model has a boundary problem at the start and at the end of this week: The operation of the system prior to and after the week under consideration influences the operation in the actual week. Since the demands - and therefore also the operation schedule - in succeeding weeks are rather similar, it has been chosen to solve this boundary problem by assuming that the simulated week is cyclic, i.e. the last time step of the week is assumed to be followed by the first one of the same week.

To find the optimal unit commitment one has to perform a load dispatching for each candidate schedule, i.e. to divide the power load between the running generators in an economically optimal way. This load dispatching procedure can be looked upon as a sub-problem to the unit commitment. It will be dealt with in Sections 4.3.4. and 4.3.5.

As described in Chapter 3 the heat production at CHP plants is found before turning to the power production in the model. As a consequence, the heat production must be left unchanged by the unit commitment and load-dispatching procedures. For back-pressure power plants this means that the power production is determined by the heat demand. Therefore, these plants do not take part in the simulation of the unit commitment and load dispatching. Instead their power production is subtracted from the power demand, whereafter the resulting demand is met by the other power-producing units.

The operational flexibility of extraction power plants is greater than that of back-pressure plants (see Chapter 2). Therefore, for time steps where extraction plants are scheduled to produce no heat they are taking part in the simulation of the unit commitment and load dispatching like condensing power plants. But for time steps where heat production is required, a certain minimum power production is necessary, too. Hence it is only left to find how much power they should produce in agreement with the limits set by the heat production.

In the remaining part of this chapter no special remarks on extraction power plants will be made: They are treated like condensing power plants, their heat production being left unchanged. As a consequence of this the heat produced by the CHP plants will not appear in the mathematical formulation of the problem given in Section 4.2.

4.2. Mathematical formulation of the power production

4.2.1. The cost function

The aim of the daily unit commitment and load dispatching is (with due regard to the reliability of supply) to minimize the total production cost. In the modelling of the problem the same philosophy is used: A cost function taking into account all relevant costs is set up together with a list of constraints that have to be observed. Among the production schedules satisfying the constraints the one with the least cost is then found by an optimization procedure.

The cost function (the object function) can be given in the following way:

$$C = f(\underline{\underline{P}},\underline{\underline{U}}) = C_{tot}(\underline{\underline{P}}) + C^s_{tot}(\underline{\underline{U}})$$

$$C_{tot}(\underline{\underline{P}}) = \Delta t \cdot \sum_{t=1}^{N_t} \sum_{i=1}^{N_g} C_i(P_{it})$$

$$C^s_{tot}(\underline{\underline{U}}) = \sum_{i=1}^{N_g} C^s_i(\underline{U}_i)$$

N_g	:	Number of production units.
N_t	:	Number of time steps.
i	:	Index for production unit.
t	:	Index for time step.
Δt	:	Length of time step.
P_{it}	:	Power production by unit i at time step t (MW).
$\underline{\underline{P}}= \{P_{it}\}$	:	Matrix representing all power productions (MW).
U_{it}	:	Zero-one variable telling whether unit i is operated (U_{it}=1) or not U_{it}=0) at time step t.
$\underline{\underline{U}}= \{U_{it}\}$	:	Matrix representing all U_{it}. $\underline{\underline{U}}$ will often be denoted "the unit commitment matrix".
$\underline{U}_i$	:	The i'th row of U.
$C=f(\underline{\underline{P}},\underline{\underline{U}})$	:	Total production costs (kr).
$C_{tot}(\underline{\underline{P}})$	:	Total running costs*) (kr).
$C^s_{tot}(\underline{\underline{U}})$	:	Total start-up costs**) (kr).
$C_i(P_{it})$	:	Running costs*) for unit i at time step t (kr/h).
$C^s_i(\underline{U}_i)$	:	Start-up costs**) for unit i (kr).

*) Fuel and O&M costs. For extraction power plants these running costs also depend on the heat power produced, but since this is unchanged by the unit commitment procedure this dependence is left out in the formulation.

**) Fixed start-up costs and start-up fuel costs.

Already at this point it can be seen that the simulation of the power production involves the solution of a mixed integer programming problem, the integer and continuous variables being represented by the $\underline{\underline{U}}$ and $\underline{\underline{P}}$ matrices, respectively. If the unit commitment matrix temporarily is assumed to be given, the task of finding the optimum $\underline{\underline{P}}$ matrix corresponds to performing a load dispatching for each time step, i.e. to find how the load could be shared among the running generators in the most economical way. This load dispatching problem is discussed in Sections 4.3.4. and 4.3.5.

Before turning to the constraints on the optimization variables, the shape of the nonlinear cost functions C_i and C_i^S will be commented upon. The way in which the costs are modelled is given in Chapter 2, but for clarity it shall be repeated here: The fuel consumption for normal production of a power plant is given by the fuel consumption at minimum load and by a continuous piecewise-linear incremental fuel consumption curve. By multiplying the fuel consumption by the price of the fuel in question, the fuel costs are obtained. The running costs C_i include not only the fuel costs but also operation and maintenance (O&M) costs which are specified as costs per MWh produced and per hour of production. Fixed operating costs (specified as costs per kW rated capacity per year) do not influence the optimum production schedule and are therefore not included in the optimization. Instead these costs are included in the optimum schedule when the total costs are evaluated.

Since the cost function $C_i(P_{it})$ for an individual generating unit must be convex, also the total running costs $C_{tot}(\underline{\underline{P}})$ for all generators in all time steps are convex with regard to the variables $\underline{\underline{P}}$. This convex nature of the cost function is essential to ensure that no local optimum exists when minimizing the costs (Section 2.3.1 in ref. 6).

The start-up costs are represented by a fixed (operating and maintenance costs) and a variable component (fuel costs). The latter depends exponentially on the time period without production. Thus, the cost for a single start-up can be represented by

$$C_{start-up} = C^{s}_{fix} + C^{s}_{var} \cdot (1-\exp(-t^{s}/T^{s}))$$

where C^{s}_{fix} is the fixed start-up costs, while C^{s}_{var} is the costs of the fuel needed to start the plant when the boiler is cold. T^{s} is the time constant for boiler cooling, and t^{s} is the time period with no production. The total start-up costs $C^{s}_{i}(\underline{U}_{i})$ for generator i may include several start-up's. At this point it should be borne in mind that the simulated time period is assumed to be cyclic, i.e. the first time step is assumed to follow the last one.

Although the nonlinear cost function $C=f(\underline{\underline{P}},\underline{\underline{U}})$ is expressed as a sum of two terms, $C_{tot}(\underline{\underline{P}})$ and $C^{s}_{tot}(\underline{\underline{U}})$, that do not depend explicitly on $\underline{\underline{U}}$ and $\underline{\underline{P}}$, respectively, the optimization problem cannot be separated into two independent problems. This is because the constraints, which will be dealt with below, link together the U's and P's.

4.2.2. The constraints

There are three different groups of constraints that must be recognized when minimizing $C = f(\underline{\underline{P}},\underline{\underline{U}})$. The constraints are grouped according to whether they involve only U's, only P's, or both the U's and the P's.

The constraints can also be grouped according to another criterion: One may distinguish constraints that are physically necessary from those that are economically reasonable. The idea of introducing this last type of constraint is to cut away from the solution space such sub-spaces that will no doubt prove too expensive. The model is so flexible that these last constraints can be bypassed if so wished.

Below the constraints will be given in both a mathematical and a more descriptive formulation. Some of the constraints are linear in the optimization variables while others are nonlinear. The indices i and t take on all integer values from one to N_g and N_t, respectively. It should be kept in mind that due to the formulation of the problem U_{it} can take on the values zero and one, only*).

1) $$\sum_{i=1}^{N_g} P_i^{max} \cdot U_{it} \geq P_{Dt} + P_{Lt} + P_{Rt}$$

P_i^{max} : rated capacity**) of generator i (MW).
P_{Dt} : power demand at time step t (MW).
P_{Lt} : power losses in the transmission system at time step t (MW).
P_{Rt} : spinning reserve at time step t (MW).

This constraint ensures that the total rated capacity of the running generators can at least match the demand, the losses, and the spinning reserve. The power losses are given by a second-order expression (as described in Section 2.8):

$$P_{Lt} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_{it} B_{ij} P_{jt} + \sum_{i=1}^{N_g} B_{oi} P_{it} + B_{oo}.$$

The spinning reserve is as shown below specified rather roughly. Therefore it is for this constraint unnecessary to express the power losses as a function of the individual power production. Instead the power losses P_{Lt} are replaced by P'_{Lt}:

$$\sum_{i=1}^{N_g} P_i^{max} \cdot U_{it} \geq P_{Dt} + P'_{Lt} + P_{Rt}$$

$$P'_{Lt} = \beta'_i \cdot P_{Dt} + \beta'_o$$

---

*) If index i stands for an extraction unit which is delivering heat at time step t, $U_{it} = 1$ a priori.

**) For extraction power plants the maximum power capacity may be reduced due to heat production (see Section 2.4).

The constants β_o' og β_1' must be chosen so that P_{Lt}' is always greater than or equal to P_L.

By excluding the P's from the constraint it is much easier to handle it in the simulations. It should be noticed that in the calculation of the costs $C_{tot}(\underline{\underline{P}})$ the more accurate expression P_{Lt} is used (see constraint number 8).

The spinning reserve P_{Rt}, which is necessary to take up forced outages of production units and load variations*), can optionally be specified in several ways:

a) $P_{Rt} = P_{Rt}^a = \max\limits_i (U_{it} \cdot P_i^{max})$

b) $P_{Rt} = P_{Rt}^b = \alpha \cdot P_{Dt}$

c) $P_{Rt} = P_{Rt}^c = \sum\limits_{i=1}^{N_g} U_{it} \cdot P_i^{max} \cdot (1-p_i)$

p_i : reliability of unit i.

d) $P_{Rt} = P_{Rt}^d = \max (P_{Rt}^a, P_{Rt}^b, P_{Rt}^c)$

If $P_{Rt} = P_{Rt}^a$ is used, there will always be sufficient spinning reserve to take over production even if the largest running generator fails. If the second formula is used, the spinning reserve is great enough to meet a relative increase of size α of the demand. The third formula involves the reliability of the power plants in a rather simple way. In principle, it is possible to formulate the constraint as an upper bound on the probability that the available production capacity is less than the load. But since the constraint is checked so many times during the simulations it would be too time consuming to perform that sort of calculation, and

*) Neither stochastic loads nor forced outages are modelled. But to find the production costs as correctly as possible, the need for spinning reserve is included in the model.

moreover, the result of the simulations could not be expected to be changed very much by introducing this refinement. The last formula for the spinning reserve chooses the maximum value of the first three ones.

2) $$\sum_{i=1}^{N_g} P_i^{min} \cdot U_{it} \leq P_{Dt} + P_{Lt}$$

P_i^{min} : minimum power production*) of generator i (MW).

This formula expresses the ability of the production from the running generators to be reduced enough to match demand and loss. As for constraint number 1, P_{Lt} is replaced by a more simple expression P_{Lt}''

$$\sum_{i=1}^{N_g} P_i^{min} \cdot U_{it} \leq P_{Dt} + P_{Lt}''$$

$$P_{Lt}'' = \beta_1'' \cdot P_{Dt} + \beta_o''$$

The constants β_1'' and β_2'' must be chosen so that P_{Lt}'' is always smaller than or equal to P_{Lt}. If this is not the case problems might arise when calculating the costs of production (where the losses are represented by P_{Lt}).

3) $$\sum_{v=t}^{t+t_1} \max\{0, U_{i(v+1)} - U_{iv}\} \leq N_i^{max}$$

$$\sum_{v=t}^{t+t_1} \max\{0, U_{iv} - U_{i(v+1)}\} \leq N_i^{max}$$

$$t_1 = t - 1 + T_i^{max}/\Delta t$$

This is an example of a constraint that is not physically necessary, but from an economical point of view reasonable. It states that neither the number of start-ups nor the number of shut-downs of unit i within a time period of length T_i^{max} may exceed N_i^{max}.

---

*) For extraction power plants the minimum power production depends on the heat production (see Section 2.4).

4) $$\sum_{i=1}^{N_g} \max\{0,\ U_{i(t+1)} - U_{it}\} \leq M^{max}$$

$$\sum_{i=1}^{N_g} \max\{0,\ U_{it} - U_{i(t+1)}\} \leq M^{max}$$

This optional constraint demands that neither the number of simultaneous start-ups nor shut-downs may exceed M^{max}.

5) $$\sum_{i=1}^{N_g} U_{it} \cdot \Delta P_i^{max} \cdot \Delta t \geq \left| P_{D(t+1)} - P_{Dt} \right|$$

ΔP_i^{max} : maximum rate of change of power production at unit i (MW/h).

This formula ensures that enough plants are running to follow the load variations. If the time step is one to two hours, this optional constraint will never be violated, since the output of a power plant can be changed by several per cent per minute. Therefore, the introduction of the maximum rate of change of power output should be viewed only as a preparation for further developments. Two other constraints involving ΔP_i^{max} that in the future could be introduced, can be expressed in the following way:

$$(U_{it} - U_{i(t-1)}) \cdot P_{it} \leq P_i^{min} + \Delta P_i^{max} \cdot \Delta t$$

$$\left| P_{it} - P_{i(t-1)} \right| \cdot U_i \cdot U_{i(t-1)} \leq \Delta P_i^{max} \cdot \Delta t$$

The first of these expressions sets an upper limit on the production at a power plant in the first time step following the start-up of the plant. The second expression simply limits the rate of change of power production for normal operation.

6) $P_{Dt} < P_{D(t+1)}$ and $U_{it} = 1 \Rightarrow U_{i(t+1)} = 1$

$P_{Dt} > P_{D(t+1)}$ and $U_{it} = 0 \Rightarrow U_{i(t+1)} = 0$

This is a mathematical formulation of the reasonable assumption that when the power demand is increasing/decreasing, no generator should be stopped/started. This optional constraint is not implemented for extraction power plants. There are three options: The constraint is considered for base-load plants only, for base- and medium-load plants, or for all power plants.

7) $U_{it} \cdot P_i^{min} \leq P_{it} \leq U_{it} \cdot P_i^{max}$

This constraint states that if the generator is operated (U_{it}=1), the power production should be within the range defined by the technical minimum and the rated capacity. Also contained in the mathematical formulation is the trivial fact that if the genrator is not operated (U_{it}=0), the power production should be zero.

8) $\sum_{i=1}^{N_g} P_{it} = P_{Dt} + P_{Lt}$

This expression is a formulation of the energy balance: The total power production must match demand and losses. A formula for the losses is given under constraint number 1 and in Section 2.8. It should be noticed that P_{Lt} is <u>not</u> replaced by P'_{Lt} or P''_{Lt}.

9) When planning the operation of a power system it is obvious that peak-load plants should be operated only for short periods, whereas base-load plants are expected to produce for longer periods. This can be formulated as an optional constraint by specifying (for the individual plants) a maximum and a minimum on the total time of operation (relative to the length of the time period simulated).

10) Another optional constraint similar to the one above but specific to base-load plants, is introduced by demanding (for the individual plants) that if a plant is started it should at least be operated for some minimum time, and that if a plant is stopped it should not be started again before some minimum time has elapsed.

11) In an optimal operation of a power system it is reasonable to expect that a plant being stopped will most often be the last one that was started ("last in - first out"). This is included as an optional constraint to the optimization. CHP plants do not have to obey this constraint.

12) In many simpler models for the operation of a power system the plants are started according to some fixed priority list. This is a rather simple model of the real world and is not normally used in the Simulachron model. But to be able to make a very fast simulation of the unit commitment (when testing other parts of the model) a priority list based on the rated capacity of the plants can optionally be used.

At the beginning of Section 4.2.2 it was mentioned that the constraints can be grouped according to two criteria. This grouping has been carried out in Table 4.1.

Table 4.1. Grouping of constraints.

Constraint number	Variables in constraint	Necessary or optional
1	U	necessary
2	U	necessary
3	U	optional
4	U	optional
5	U	optional
6	U	optional
7	P,(U)	necessary
8	P	necessary
9	U	optional
10	U	optional
11	U	optional
12	U	optional

4.3. Simulation of the power production

4.3.1. Introduction

The costs of power production are given by the function $C=f(\underline{\underline{P}},\underline{\underline{U}})$ (see Section 4.2.1). It depends on both continuous ($\underline{\underline{P}}$) and zero-one variables ($\underline{\underline{U}}$). The U's specify the unit commitment schedule, and the P's give the level of production for each power plant. The costs can be separated into two terms: The running costs $C_{tot}(\underline{\underline{P}})$ that depend only on the P's, and the start-up costs $C^S_{tot}(\underline{\underline{U}})$ that depend only on the U's. However, the simulation of the power production cannot be separated into two independent simulations, since the constraints link together the U's and P's.

The Branch-and-Bound approach discussed in general terms in Appendix A is used to solve the unit commitment problem formulated in Section 4.2.

The load-dispatching problem, which can be considered a subproblem to the unit commitment, is solved by the coordination method (see Sections 4.3.4 and 4.3.5).

4.3.2. Unit commitment

In the $\underline{\underline{U}}$-matrix (integer variables) the rows and columns correspond to production units and time steps, respectively. The order in which the U's are fixed in the decision tree (see Appendix A) has been chosen so that U_{it} is fixed at level $t+(i-1)\cdot N_t$ where N_t is the number of time steps. This means that the zero-one variables in $\underline{\underline{U}}$ are fixed row by row.

Furthermore, it has been chosen to sort the power plants according to their rated capacity so that the largest plants are committed to produce (or not produce) at the upper part of the decision tree. This choice is made because only small changes of the lower bound on the costs can be expected when the integer variable associated with a small plant is fixed to zero or one, whereas greater increases of the lower bound (in many cases) can be expected when the integer variable for a larger plant is fixed to the non-optimal value. In this way it is possible already in the upper part of the decision tree to discard whole subsets of solutions because the lower bound on the costs is greater than the costs of the hitherto best solution.

The strategy used to determine the order in which the open nodes should be treated will be discussed next. Since a minimum cost solution is sought it seems reasonable always to choose the node with the least lower bound on the costs. On the other hand, as the lower bounds are increasing down through the decision tree, this will concentrate the computations at the upper part of the tree and thus create many open nodes that have to be stored. It has been shown advantageous to overcome this problem by introducing a modified lower bound:

$C'_{LB} = C_{LB} \cdot (1-\alpha_{LB} \cdot j/n)$

j : index giving the level in the decision tree.

n : total number of levels.

α_{LB} : constant (0.01-0.1).

C_{LB} : lower bound (kr).

C'_{LB} : modified lower bound (kr).

C'_{LB} is used instead of C_{LB} only when deciding which node to treat.

To reduce the number of open nodes that have to be stored a further refinement is introduced: If the relative change in the modified lower bound, when following a branch from a node to its descendant, is smaller than ε_{LB} the descendant is further partitioned, although it might not be the node with the smallest modified lower bound. It has been found that a value of ε_{LB} on the order of 0.001 is optimal.

When it has been decided which node to partition next, one also needs a strategy to determine, on which of the two descendants one should concentrate, the other one being left open for later computations. At this point a distinction is made between base, medium, and peak-load power plants in order to treat these three groups of plants differently. When a set of solutions is separated into two subsets (see Appendix A) by replacing $U_{it} = 2$ by $U_{it} = 0$ (no production) and $U_{it} = 1$ (production), where plant i might be a base, medium, or peak-load plant, the subset corresponding to U_{it} equal to U_{base}, U_{medium}, or U_{peak} is considered first. To follow a path that most probably will lead to a low-cost solution $U_{base} = 1$ and $U_{peak} = 0$ should be used. Test runs with Simulachron have confirmed this. The value for U_{medium} has no influence on the computation time. It should be emphasized that this distinction between base-, medium-, and peak-load plants does not influence the optimum solution. It is introduced only in order to find the optimum as fast as possible.

4.3.3. Constraints on the unit commitment

The constraints that must be observed during the optimization are given in Section 4.2.2. Table 4.1 shows that all constraints, except numbers 7 and 8, involve only the zero-one variables. These constraints are treated as described in Appendix A. That is they are used both in a "soft" version at levels above the bottom level of the decision tree and in a "hard" version at the bottom level. The hard constraints are programmed according to the formulation in Section 4.2.2, whereas necessary simplifications have been introduced to give the soft constraints. It should be borne in mind that the idea of soft constraints implies that if a soft constraint is violated, the corresponding hard constraints is also violated.

4.3.4. Nodes at the bottom level of the decision tree

The only constraints that involve the P's are numbers 7 and 8 (see Section 4.2.2). Therefore, these two constraints are the only ones to observe when performing the load dispatching that gives the running costs associated with the unit commitment schedule for the node considered.

If the node is at the bottom level of the decision tree it is known for each time step which plants are operated ($U_{it} = 1$) and which are not ($U_{it} = 0$). Therefore, the load dispatching can be performed rather easily. The fuel consumption of the plants is given by the fuel consumption at minimum load and by a piecewise linear incremental fuel consumption curve (see Section 2.2). Variable operating and maintenance costs are also included.

Since all the integer variables are fixed to zero or one, the start-up costs can also easily be found. These costs together with the running costs found by the load dispatching give the total costs for the node considered.

Constraint number 7 (see Section 4.2.2) is a formal representation of the trivial fact that the power production should be zero ($P_{it} = 0$) if plant i is not operated ($U_{it} = 0$), and that the power production should be within the range of operation ($P_i^{min} \leq P_{it} \leq P_i^{max}$) if the plant is scheduled for operation ($U_{it} = 1$). Constraint number 8 states that the total power production must match demand and losses specified by the B-constants (see Section 2.8).

In the following discussion of the method used for the load dispatching the time index t is omitted, and only generators committed to produce are considered.

The load dispatching can be formulated as follows:

$$\text{Minimize} \quad f = \sum_{i=1}^{n} C_i(P_i)$$

$$\text{subject to} \quad \sum_{i=1}^{n} P_i = P_D + P_L$$

$$\text{and} \quad P_i^{min} \leq P_i \leq P_i^{max} \quad , \; i=1,2, \ldots\ldots, n$$

$$\text{where} \quad P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{oi} P_i + B_{oo}$$

$C_i(P_i)$: running costs (fuel, O&M) for plant i (kr/h).
P_i : power production at plant i (MW).
P_i^{min} : minimum power production at plant i (MW).
P_i^{max} : rated capacity of plant i (MW).
P_D : power demand (MW).
P_L : power loss (MW).
B_{ij}, B_{oi}, B_{oo} : B-constants (MW^{-1}, MW/MW, and MW, respectively).
n : number of plants committed to production.

If for a while the constraints $P_i^{min} \leq P_i \leq P_i^{max}$, i = 1,2,..,n are disregarded the optimum can be found as the minimum of the Lagrange function £ (ref. 6, 7, and 8):

$$£ = \sum_{i=1}^{n} C_i(P_i) - \lambda \cdot \left(\sum_{i=1}^{n} P_i - P_D - P_L \right).$$

By introducing the Lagrange multiplier λ the constrained optimization problem has been changed into an unconstrained one. The optimum is found as the solution to the following equations:

$$\partial\pounds/\partial P_i = dC_i(P_i)/dP_i - \lambda\cdot(1-\partial P_L/\partial P_i)=0, \quad i=1,2,\ldots,n$$

and

$$\partial\pounds/\partial\lambda = -\sum_{i=1}^{n} P_i + P_D + P_L = 0.$$

The latter expression can be recognized as identical to constraint 8.

The incremental transmission losses ITL_i can be written in the following way:

$$ITL_i = \partial P_L/\partial P_i = 2\cdot\sum_{j=1}^{n} B_{ij}P_j + B_{oi}, \quad i=1,2,\ldots,n \quad .$$

The above equations for the optimum can be rewritten as:

$$\lambda = (dC_i(P_i)/dP_i)\ /\ (1 - ITL_i) \quad , \quad i = 1,2,\ldots,n$$

and

$$\sum_{i=1}^{n} P_i = P_D + P_L \ .$$

These equations are generally called the coordination equations (ref. 4 and 7). They cannot be solved analytically. Instead the solution is found by the iterative procedure shown in Fig. 4.1 (the coordination method).

Since $dC_i(P_i)/dP_i$ is the marginal cost of power load at plant number i, $(dC_i(P_i)/dP_i)/(1-ITL_i)$ is the marginal cost at plant i if the power demand increases by 1 MW*). Then it is seen that

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*) The power load tells how much is produced at the plants, whereas the power demand tells how much electricity the consumers are using. The power losses constitute the difference between load and demand.

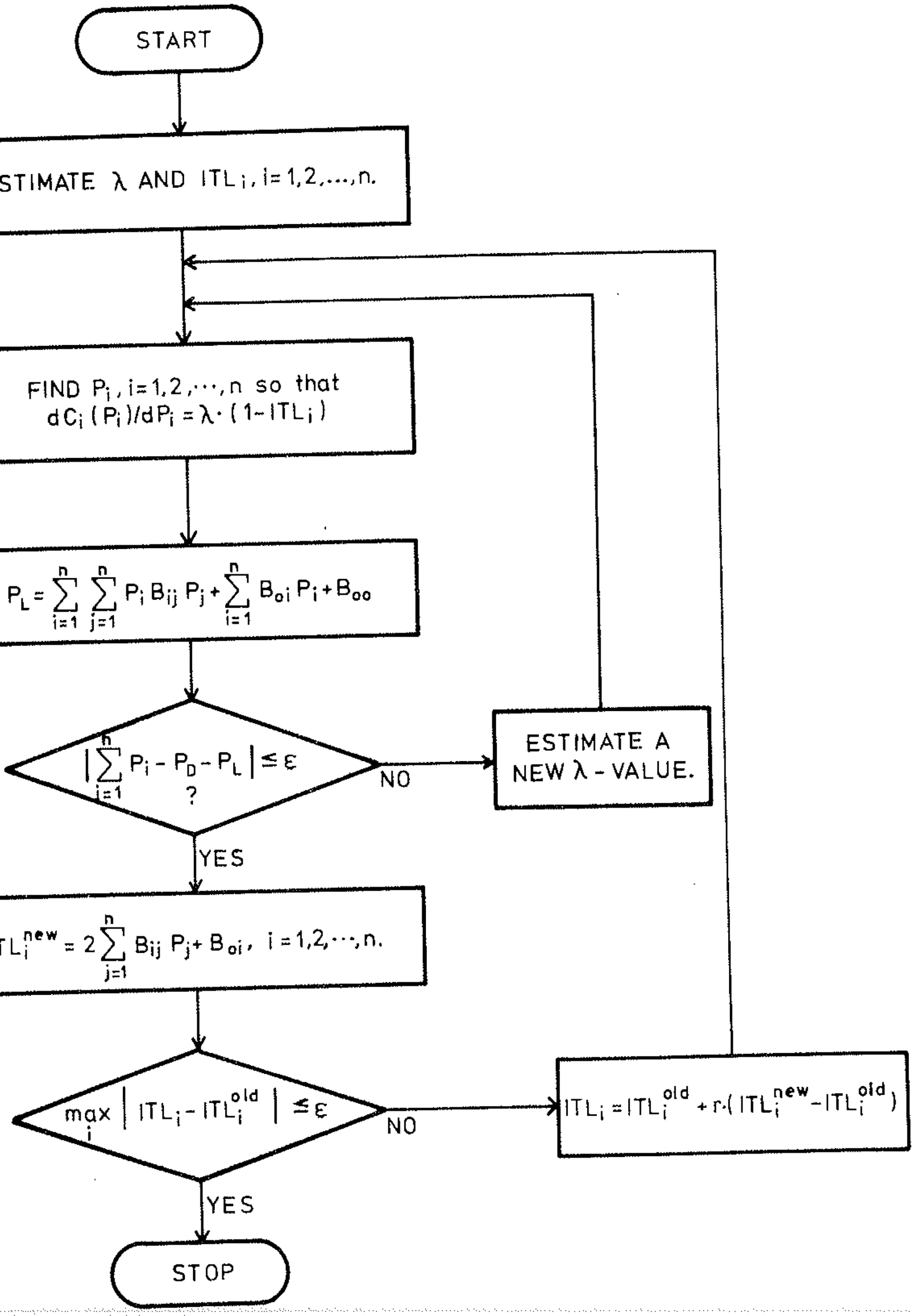

Fig. 4.1. Iterative procedure used for load dispatching at the lowest level of the decision tree. r is a relaxation coefficient ($o<r\leqslant 1$).

the coordination equations simply state that all plants should be operated at the same marginal cost (λ) of power demand.

The constraints $P_i^{min} \leqslant P_i \leqslant P_i^{max}$, $i = 1,2,\ldots,n$ are taken into account in the following way: If $\lambda \cdot (1-ITL_i)$ is smaller than $dC_i(P_i^{min})/dP_i$ or greater than $dC_i(P_i^{max})/dP_i$ P_i is set to P_i^{min} or P_i^{max}, respectively.

4.3.5. Nodes above the bottom level of the decision tree

For nodes above the bottom level of the decision tree the computations performed to find a lower bound on the costs for the set of solutions associated with the node differ somewhat from the computations for nodes at the lowest level.

Start-up costs are included only for generators that have been scheduled for all time steps, i.e. there are no 2's in the corresponding row of the U-matrix. A load dispatching between units for which U_{it} equals 1 or 2 is performed for each time step t. Naturally power plants with $U_{it} = 0$ do not take part in this, whereas plants with $U_{it} = 1$ are included. Production units with $U_{it} = 2$ (i.e. scheduled neither for production nor shut down) are replaced by a "lower bound version" of the unit. This lower bound version is chosen so that it is cheaper and more flexible to operate than is the correct version of the plant.

By letting the lower bound version instead of the correct version of the plant take part in the load dispatching it is ensured that the running costs form a true lower bound on the costs (together with the start-up costs).

It is assumed that the lower bound version of a plant can be operated at any level of production from zero to rated capacity. Figure 4.2 shows the variable costs (fuel, operating, and maintenance costs) for the lower bound version together with the correct cost curve. It is seen that the lower bound cost curve is a convex curve that starts at the origin and throughout the range of operation lies as close as possible to the correct cost curve without ever rising above this. For simplicity it has been chosen to represent the lower-bound version of the plants by a linear and slightly increasing incremental cost curve, corresponding to a second-order cost curve. The algorithm (given below) used to perform the load-dispatching demands an increasing incremental cost curve.

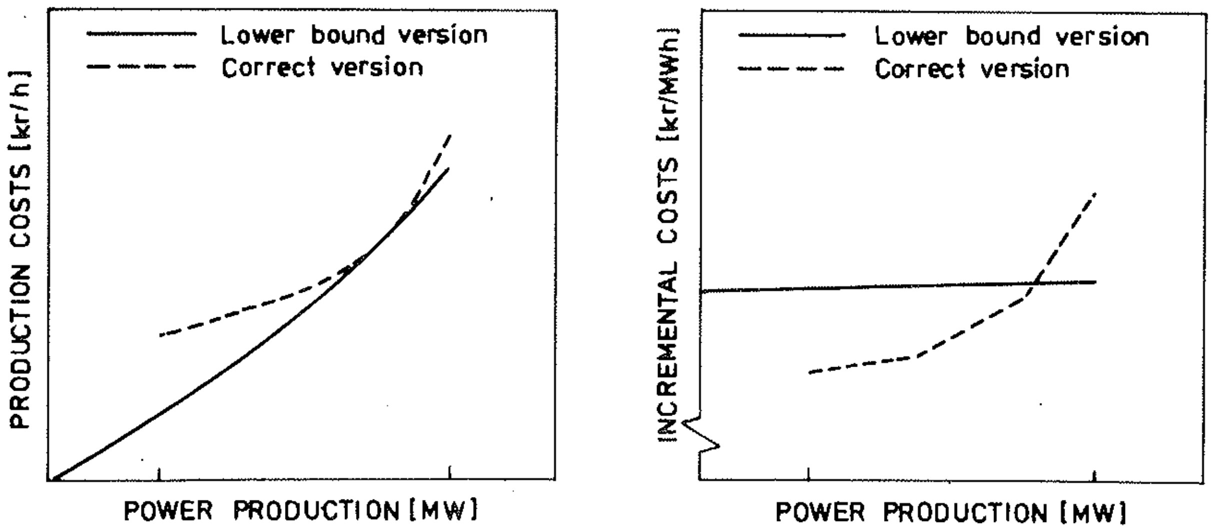

Fig. 4.2. Example of variable costs (fuel, operating, and maintenance). Lower-bound version and correct version. The non-linearities are exaggerated for clarity.

The load-dispatching performed to find lower bounds on the costs associated with nodes above the last level of the decision tree could, in principle, follow the procedure given for nodes at the bottom level. But in order to save computer time a simplification is introduced: The power losses are not specified by the B-constants. Instead they are given by the following simpler expression (the time index t still being omitted):

$$P_L^{LB} = \gamma_1 \cdot P_D + \gamma_0$$

The constants γ_1 and γ_0 must be chosen so that P_L^{LB} is always smaller than or equal to P_L (since a lower bound on the costs is sought).

Specifying the power losses by P_L^{LB} the load dispatching can be formulated as:

$$\text{Minimize } f = \sum_{i=1}^{n} C_i^{LB}(P_i)$$

$$\text{subject to} \quad \sum_{i=1}^{n} P_i = P_D + P_L^{LB}$$

$$\text{and} \quad P_i^{LBmin} \leq P_i \leq P_i^{max} \quad , \; i=1,2,\ldots,n$$

$$\text{where} \quad P_L^{LB} = \gamma_1 \cdot P_D + \gamma_0 .$$

The new variables introduced are:

$C_i^{LB}(P_i)$: running costs (fuel, O&M) for plant i (kr/h).
If $U_i = 1$, C_i^{LB} is identical to C_i.
If $U_i = 2$, C_i^{LB} gives the costs for the lower bound version of plant i.

P_i^{LBmin} : minimum power production at plant i (MW).
If $U_i = 1$, P_i^{LBmin} equals P_i^{min}.
If $U_i = 2$, P_i^{LBmin} equals zero.

In the formula giving P_L^{LB} the demand P_D can be replaced by the productions:

$$P_L^{LB} = \gamma_3 \cdot \sum_{i=1}^{n} P_i + \gamma_2$$

where $\gamma_2 = \gamma_0/(1+\gamma_1)$

and $\gamma_3 = \gamma_1/(1+\gamma_1)$.

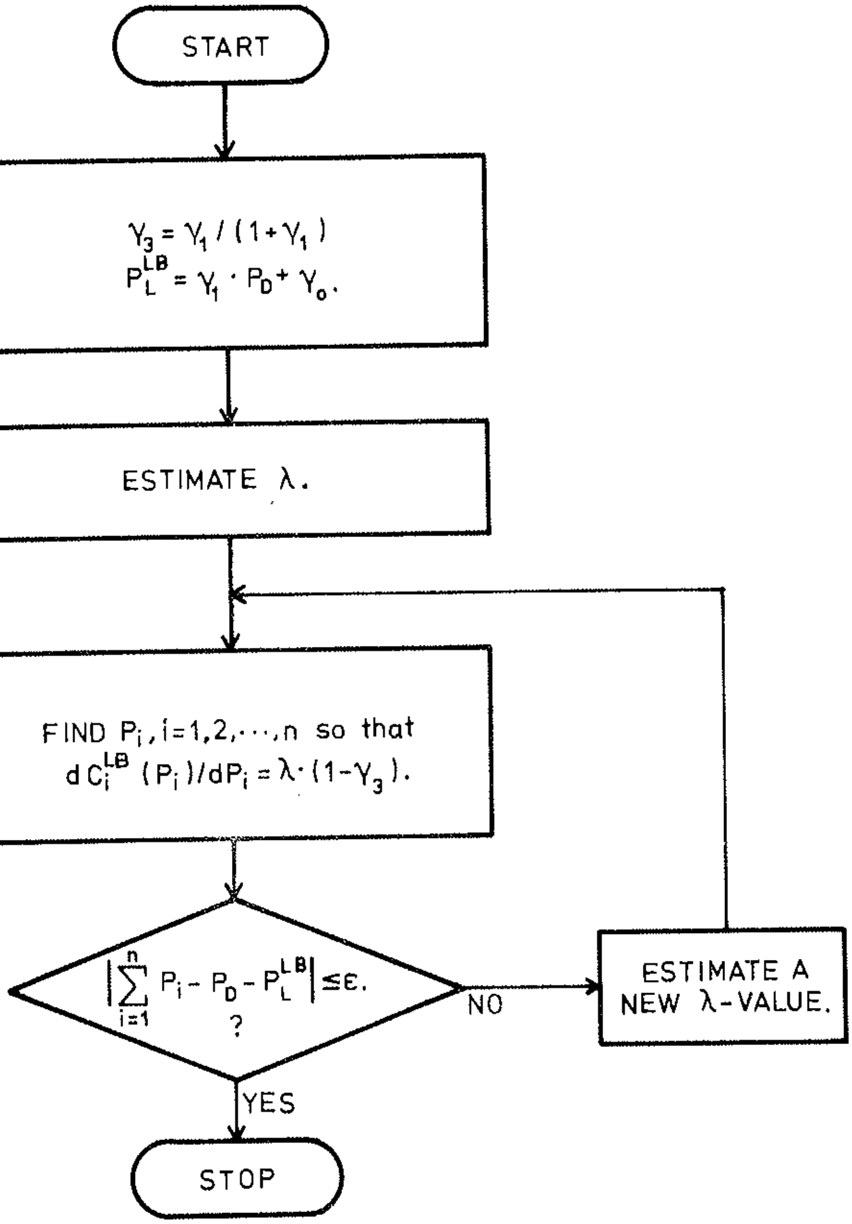

Fig. 4.3. Iterative procedure used for load dispatching above the lowest level of the decision tree.

In this formulation the incremental transmission losses are independent of the productions:

$$ITL_i = \partial P_L^{LB}/\partial P_i = \gamma_3 \quad .$$

Therefore, the coordination equations have the following simple form:

$$\lambda = (dC_i^{LB}(P_i)/dP_i) \;/\; (1 - \gamma_3) \quad , \quad i=1,2,\ldots,n$$

$$\sum_{i=i}^{n} P_i = P_D + P_L^{LB}$$

The flow chart in Fig. 4.3 gives the iterative procedure used to perform the load dispatching. By comparing Fig. 4.1 and 4.3 it is seen that the introduction of P_L^{LB} has removed the outer iteration loop.

4.4. Unit commitment as a series of sub-optimizations

4.4.1. Introduction

In Section 4.3 the unit commitment schedule for say one week was found in a single overall optimization using Branch-and-Bound techniques. With a time step of length say 2 hours, the number of integer variables will therefore be rather large, and the computer simulations might be too time consuming unless the demanded accuracy of the solution is reduced (ε-optimality, see Appendix A). Therefore to bring down the number of variables that are optimized simultaneously the computations are decoupled in a series of sub-optimizations. The Branch-and-Bound approach is also used for these optimizations. The Simulachron model is so flexible that, through the input, the user is able to specify whether a total simulation is wanted, or whether one of the decoupling methods described below should be used. Figure 4.4 shows the various sub-optimizations that are allowed for in the model.

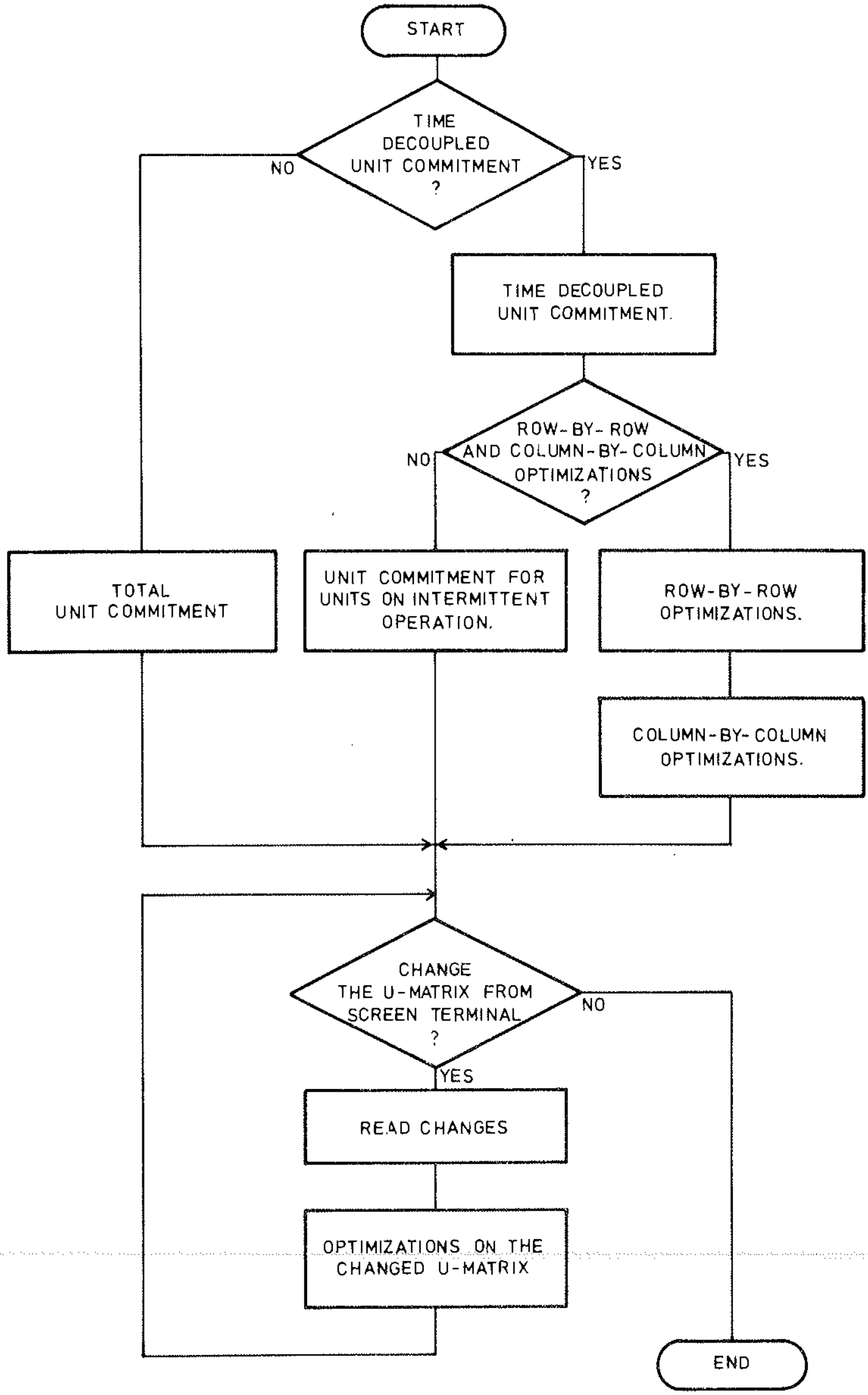

Fig. 4.4. Sub-optimizations in the Simulachron model.

4.4.2. Time-decoupled unit commitment

In a time-decoupled unit commitment the time steps are considered one at a time. Constraints that contain variables from more than one time step cannot be considered. Neither can start-up costs. Therefore, one cannot hope to get an optimal overall unit commitment schedule by combining the results of the time decoupled optimizations. Instead, the time-decoupled computations which are performed very fast should be viewed upon as an initialization to get a first guess of the solution before one of the two methods given in Sections 4.4.3 and 4.4.4 are used.

4.4.3. A renewed unit commitment for units on intermittent operation

In the unit commitment matrix $\underline{\underline{U}}$ (defined in Section 4.2.1) the rows and columns correspond to production units and time steps, respectively. Therefore a $\underline{\underline{U}}$-matrix can be set up column by column from the time-decoupled solutions discussed above. In this matrix some units are scheduled for operation in all time steps (the corresponding row contains only ones) while others are scheduled for no operation at all (only zeros in the row). The rest of the generators are committed to be operated intermittently (both zeros and ones in the row).

A renewed unit commitment is performed for these generators for all time steps. Mathematically this is done by replacing rows holding both ones and zeros by rows of twos (the number two indicates that no decision - zero or one - has yet been taken, see Appendix A). The Branch-and-Bound method is then used to find the optimum values for these variables. The total cost function and all constraints (see Sections 4.2.1 and 4.2.2, respectively) are used in this optimization.

It seems reasonable to expect that the optimum solution for the total unit commitment problem discussed in Section 4.3 is a member of the subset of solutions considered here. This is because the introduction of start-up costs that were not included in the decoupled optimizations can be expected to reduce or at least not increase the number of start-ups. Therefore, the unit commitment schedule for plants that in the time-decoupled optimizations were committed to be operated all the time (or not operated at all) would not have been changed if these plants had taken part in the last optimization.

4.4.4. Row-by-row and column-by-column optimizations

Starting from the result of the time-decoupled unit commitment a series of sub-optimizations can be performed. In the row-by-row optimizations the rows of the U-matrix are one by one replaced by a row of twos, whereafter, the optimum values (zero or one) for the variables in this row are found by Branch-and-Bound techniques. When the last row of the matrix has been treated, the computations start all over again with row number one, row number two, and so on. This goes on until the improvement of the total costs (i.e. for all generators) is small enough. This improvement is defined as the difference between the total costs found when optimizing a certain row, and the total costs found the previous time this row was considered.

The column-by-column optimizations are succeeding the row-by-row computations. The only difference is that now the U-matrix is scanned column-by-column.

It is possible (not shown in Fig. 4.4) to perform an iteration around the combined row-by-row and column-by-column sub-optimizations. But test-runs of the Simulachron model have shown that no further reduction of the costs is obtained by this - unless the demanded accuracy of the solutions is increased (ε-optimality, see Appendix A).

4.4.5. Unit commitment changed from screen terminal

Before finishing the simulations the user has a possibility to change the U-matrix from the computer terminal (see Fig. 4.4). Any variable in the matrix can be changed to any of the values zero, one or two*). If the new matrix contains one or more twos the optimum value (zero or one) for these are found by the model. This optimization is performed as one total optimization, i.e. no sub-optimizations as described in Sections 4.4.2, 4.4.3, and 4.4.4 are performed. If there are no twos in the matrix only the total costs are found (provided that no constraints are violated).

The possibility of changing the unit commitment matrix from the terminal can be used to find the costs associated with some specific unit commitment schedule, or to find the optimum operation strategy for some plants (or time steps) when the operation for other plants (or time steps) is given by the user.

4.5. Evaluation of the Branch-and-Bound technique

The Branch-and-Bound technique has shown its strength and flexibility in the simulation of the power production. The huge number of possible solutions to the unit commitment problem is coped with by the technique, and constraints on the optimization can easily be handled. Moreover the Branch-and-Bound approach allows the user to decide whether the optional constraints should be considered or not (see Section 4.2.2).

As could be expected, test-runs of the Simulachron model have shown that the number of nodes in the decision tree that are considered during a simulation depends very much on the ε-value used to define the accuracy of the solution (ε-optimality, see Appendix A). Moreover the computer time needed for a simulation can vary drastically for only minor changes of the input to the model. The cause of this is that the number of nodes that must be checked before the optimum is found, can be strongly reduced if a cheap solution at the bottom of the solution tree is found at an early stage of the optimization.

*) For extraction power plants $U = 0$ is illegal for time steps with heat production.

The optional constraints that are introduced for economical reasons (see Section 4.2.2) do not always bring about a reduction of the computer time, and in many cases they could just as well be excluded. The reason for this is that when the computations for a node are finished, the node with the smallest (modified) lower bound on the costs will be considered next (see Section 4.3.2). Therefore the simulations will be concentrated on the more economical subsets of solutions, and the constraints introduced for economical reasons will seldom be violated.

As described in Appendix A whole subsets of solutions can be discarded if the associated lower bound on the costs is greater than the costs of the hitherto best solution. Therefore it is essential to the effectiveness of the Branch-and-Bound approach that the lower bound on the costs for a set of solutions is close to the real costs for these solutions. If the approximations used to find the lower bound are too coarse, the lower bound will be unnecessarily small.

To get an estimate of the difference between the variable costs for the correct version and the lower bound version of the power plants (see Fig. 4.2) an examination has been performed for 33 power units belonging to the ELSAM power company that serves all of Denmark west of the Great Belt. For the lower bound version of the power plants (defined in Section 4.3.5) the incremental cost curve is linear and only slightly increasing. Hence, when performing a load dispatching for power plants represented by their lower bound version the plants will most often be operated at zero or maximum production while only one plant is operated at some intermediate level. If no production takes place, the lower bound version gives the correct costs (i.e. zero costs). When operating all the above mentioned 33 power plants at maximum production the difference between correct and lower bound costs is two per cent. This means that the variable costs for unscheduled units ($U_{it} = 2$) as estimated by the load dispatching procedure on an average cannot be expected to be larger than some 98 per cent of the total cost if the units in question had been scheduled for production ($U_{it} = 1$).

For the two examples given in Tables 6.15 and 6.16 the start-up costs amount to 0.6 and 1.1% of the total costs, respectively. Hence, at the top of the decision tree (where all power plants in all time steps are represented by their lower bound version and no start-up costs are included), the estimated lower bound on the total costs (variable costs and start-up costs) can be expected to be some three per cent lower than the costs for the optimum solution.

The discussion of the above paragraph shows that many paths down through the decision tree will not be stopped at the upper part of the tree. Instead they must be followed through branchings to the lower part of the tree before the lower bounds become so great that the nodes can be deleted.

5. HEAT PRODUCTION

5.1 Introduction

As discussed in Chapter 3 the simulations of heat and power production are performed separately but linked together by the price of the co-produced power. This is done to keep down the number of variables that have to be optimized at the same time.

The district heating areas are non-interacting and therefore considered one at a time in the computations. A district heating area is defined as a system consisting of a district heating grid with a heat demand and heat production system. This production system might comprise CHP plants, district heating boilers, and refuse incineration plants. A heat storage facility for day-to-day storage of hot water is optional, too. The representation of the production units and storage facilities is given in Chapter 2. The district heating grid is taken into account by specifying - for the individual heat producing units - the relative losses in the transmission of heat to the consumers. The heat demand is given as a deterministic time series. The same relative time series is used for all heating areas, but of course the absolute magnitude of the demand varies from one heating area to another.

The aim of the simulations of the heat production is to minimize the total costs. Therefore, the value of the co-produced electricity from CHP plants should also be taken into account. The price of this electricity is given as a time series that is found in the simulation of the power production during which the heat production schedule is not changed. Therefore, as described in Chapter 3, an iteration around the heat production part and power production part of the simulations must be performed until the situation is stable. If no heat storage facility is present in any district heating area it is sufficient to make one or two simulations of the heat and power production. That is, one has to feed back the marginal power costs one time at the most.

For district heating areas with heat storage the time steps cannot be considered separately since the storage facility makes it possible to transfer heat production from one time step to another. For such heating areas the simulations are carried out at two levels: In an outer iteration the optimal operation of the storage is found, and for each step in this iteration the heat load is dispatched in the most economical way among the production units at the inner level.

As in the simulation of the power production, the time period considered is assumed to be cyclic, i.e. the first time step follows the last one. Therefore it is unnecessary to specify the energy contents of the storage facility prior to or after the simulated period.

In the remaining part of Chapter 5 the modelling of the heat producing plants will be given in Section 5.2, while the heat load dispatching is discussed in Section 5.3. The modelling of the operation of a heat storage facility is considered in Section 5.4.

5.2. The heat producing plants

5.2.1. Introduction

To avoid the unit commitment problem in connection with the simulation of the heat production, start-up costs at CHP plants are not included in the computations. Therefore, the time steps can be considered one at the time as far as heat load dispatching is concerned.

In the procedure used for heat load dispatching, CHP plants are temporarily allowed to be operated below their technical minimum. If a CHP plant at the optimum solution for some time steps produces below its technical minimum, a renewed optimization is performed during which this plant is not at all allowed to be operated for these time steps.

5.2.2. Back-pressure power plants

Figure 5.1 shows the range of operation for a back-pressure power plant (see Figs. 2.2 and 2.3). Also shown are the incremental and the total cost curves. The dashed lines correspond to operation below the technical minimum. To keep the cost

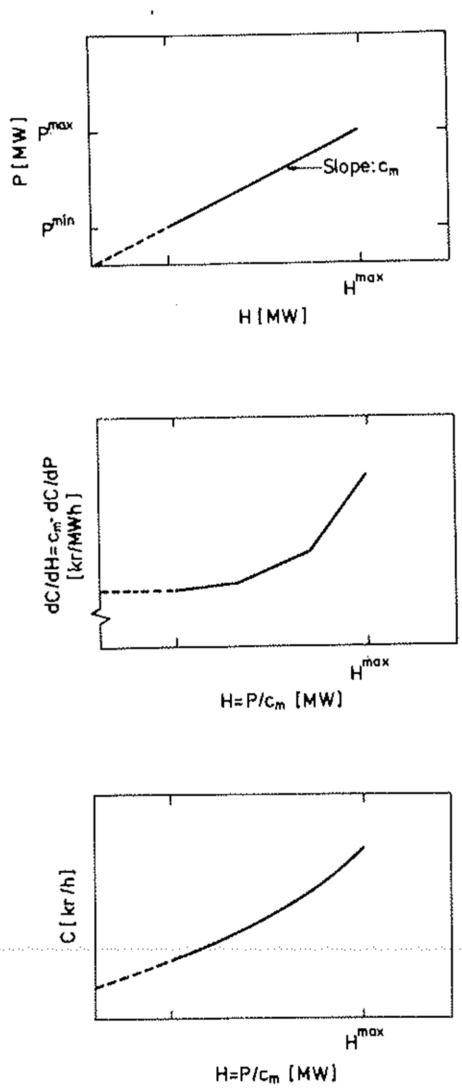

Fig. 5.1. Back-pressure power plant. Range of operation, incremental cost curve, and total cost curve.
Dashed lines: operation below technical minimum.

curve convex during the optimizations (to avoid local minima) a positive cost is assumed for zero production. When the optimum solution has been found the costs for no production are changed to zero.

The variables introduced in Fig. 5.1 are:

- H : Heat production (MW).
- H^{max}: Maximum heat production (MW).
- P : Power production (MW). $P = c_m \cdot H$ for a back pressure plant.
- P^{min}: Minimum power production (MW).
- P^{max}: Maximum power production (MW).
- C : Running costs (kr/h). Includes fuel and O&M costs.
- c_m : Back-pressure constant (MW/MW).

Elsewhere these variables might be used with a subscript i to identify the plant.

5.2.3. Extraction power plants

The range of operation for an extraction power plant is shown in Fig. 5.2. As described in Section 2.4 (see Fig. 2.11) lines with slope $-c_v$ are equi-cost curves. Therefore, the running costs C can be interpreted as a function of $P_o = P + c_v \cdot H$. If the abscissa H is replaced by P_o the two lower graphs of Fig. 5.1 can represent the dependency of C on P_o.

New variables in Fig. 5.2 are:

- c_v : Absolute value of the slope of equi-cost curves (MW/MW).
- P_o : Equivalent power production (MW). $P_o = P+c_v \cdot H$.
- P_o^{min} : Minimum equivalent power production (MW).
- P_o^{max} : Maximum equivalent power production (MW).

As mentioned above, the co-produced electricity is assumed to be sold to the surrounding power system at a price that does not depend on the amount of electricity produced by CHP plants in

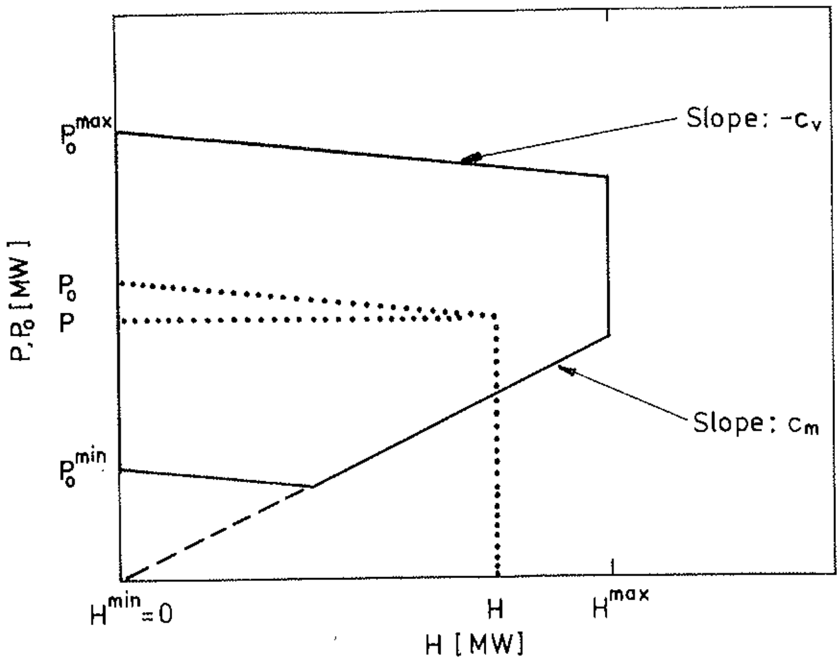

Fig. 5.2. Extraction power plant. Range of operation. Dashed line: Operation below technical minimum.

the district heating area considered. Therefore, for a production schedule to be optimal all extraction power plants should be operated at a marginal cost of power production equal to the price at which the co-produced power is sold - unless producing at minimum or maximum power. This can be utilized to eliminate the level of power production at extraction plants from the set of optimization variables.

To do this a distinction must be made between the following four cases:

1) Minimum power production:

$$c_p < (dC/dP_o)_{P_o=P_o^{min}}$$

2) Intermediate power production:

$$(dC/dP_o)_{P_o=P_o^{min}} \leq c_p < (dC/dP_o)_{P_o=P_o'}$$

3) Intermediate power production:

$$(dC/dP_o)_{P_o=P_o'} \leq C_p < (dC/dP_o)_{P_o=P_o^{max}}$$

4) Maximum power production:

$$(dC/dP_o)_{P_o=P_o^{max}} \leq C_p$$

New variables have been introduced:

$P_o' = (c_m+c_v)\cdot H^{max}$ (see Fig. 5.3).

C_p : Price at which the co-produced electricity is sold (kr/MWh). The time dependency of C_p has been omitted for clarity.

For each of the four cases the power production is given by the heat production as shown in Fig. 5.3. In cases two and three the lines with slope $-c_v$ are placed so that $\partial C/\partial P = C_p$ for points on the lines.

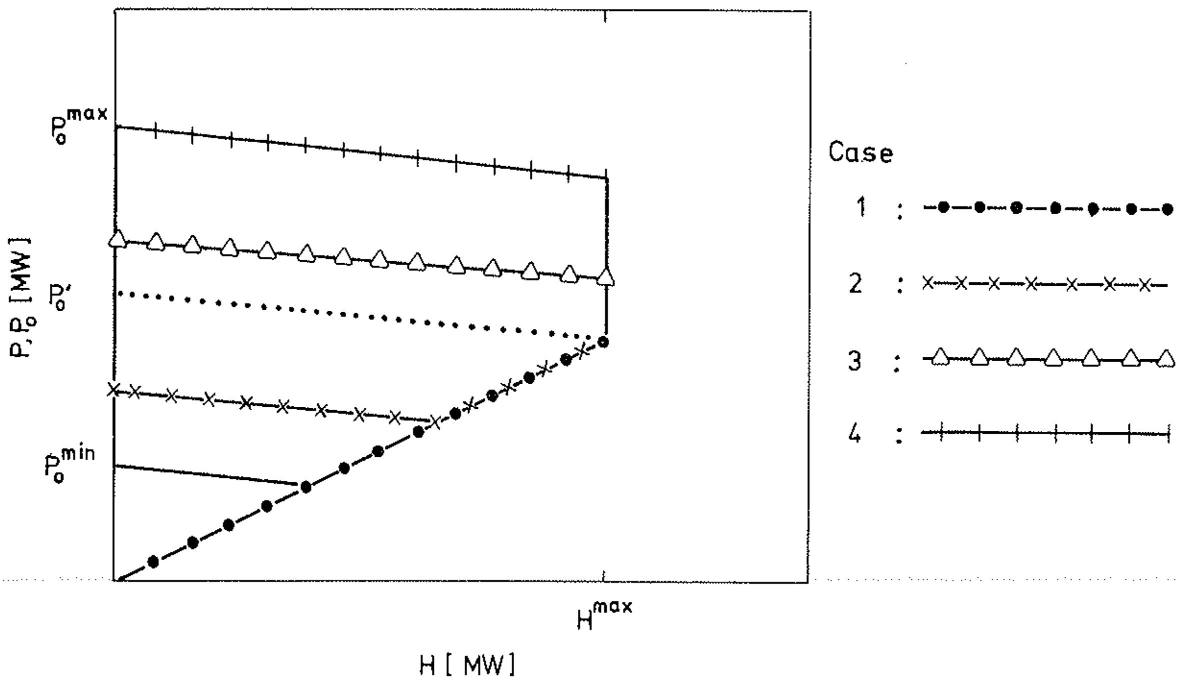

Fig. 5.3. Extraction power plant. When the heat production is given, the power production is found at one of the indicated lines.

5.2.4. District heating boilers

District heating boilers and refuse incineration plants are modelled in a rather simplified way in the simulations: Their specific production costs are considered constant throughout the total range of operation.

5.3. Heat load dispatching

5.3.1. Introduction

In this Section 5.3 the operation schedule for a possible heat storage facility is assumed to be fixed (although it might not be the optimum schedule). Therefore the load as seen from the production system might include the power (positive or negative) used to charge the storage. The system is shown in Fig. 5.4.

5.3.2. Method of heat load dispatching

For a production schedule to be optimal all heat-producing units should be operated at the same marginal cost of heat production - unless operated at minimum or maximum production. To take into account the different losses in the heat transmission system this marginal cost should be understood as marginal cost per unit of energy delivered to the consumers.

The marginal costs also include the income from selling the co-produced electricity:

$$\lambda_h = (dC/dH - C_p \cdot dP/dH)/\eta$$

η : Effectivity (1-losses) in heat transmission from plant to the consumers (p.u.).

λ_h : Marginal cost of delivered heat energy (kr/MWh).

In this formula the time index t and the plant index i are omitted. Moreover, it should be borne in mind that the running costs of extraction plants can be considered a function of

$P_o = P + c_v \cdot H$. Therefore, for these plants the term dC/dH should be interpreted as $(dC/dP_o) \cdot (dP_o/dH)$.

For the various types of plants the expression for λ_h can be changed in the following ways:

1) Back-pressure power plants:

$$\lambda_h = c_m \cdot (dC/dP - C_p)/\eta$$

2) Extraction power plants:
If the point of operation is on the back-pressure line (see Fig. 5.3):

$$\lambda_h = ((c_m + c_v) \cdot dC/dP_o - c_m \cdot C_p)/\eta$$

Else:

$$\lambda_h = c_v \cdot C_p/\eta$$

3) District heating boilers and refuse incineration plants:

$$\lambda_h = (dC/dH)/\eta$$

Since all plants should be operated at the same value of λ_h (unless operated at minimum or maximum production) the total heat delivered to the consumers can be expressed as a function of λ_h:

$$H_{tot}(\lambda_h) = \sum_{i=1}^{N_h} \eta_i \cdot H_i(\lambda_h)$$

N_h : Number of heat producing units in the district heating area.
i : Index for production unit.
H_i : Heat produced at unit i (MW).
H_{tot}: Total heat delivered to the consumers (MW).

(The functions $H_i(\lambda_h)$ depend implicitly on the price C_p of the co-produced electricity, and since C_p varies with the time the shape of the function $H_{tot}(\lambda_h)$ also depends on the time. Therefore a new function $H_{tot}(\lambda_h)$ must be set up for each time step).

In the Simulachron model the function $H_{tot}(\lambda_h)$ is used in an iteration*) to find that λ_h which gives a total delivered heat power H_{tot} that equals the consumers' demand (plus heat used to charge a possible storage unit). From this λ_h-value the productions $H_i(\lambda_h)$ of the individual plants are found.

5.4. Optimum operation of a heat storage facility.

5.4.1. Introduction

In Section 5.4 only the operation of a possible heat storage facility will be dealt with. The sub-problem of heat load dispatching has been discussed in Section 5.3.

Since the simulation of long time periods in the model is broken down into simulations of, for instance, weeks that are considered non-interacting, only day-to-day heat storage is covered by the model.

5.4.2. Formulation of the problem

The simulated system is shown in Figure 5.4. The heat energy follows the lines (in the directions indicated by the arrows) from the heat-producing plants to the storage facility and to the consumers. The effectivities (1-losses) of the individual transmission lines are also shown.

The plants with indices 1,2,...,k directly connected to the storage unit will typically be coal- or oil-fired CHP plants or refuse incineration plants, while plants k+1, k+2,...,N_h will be district heating boilers.

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*) This iteration is performed rather fast since the function $H_{tot}(\lambda_h)$ is piecewise linear. It is tabulated only once for each time step.

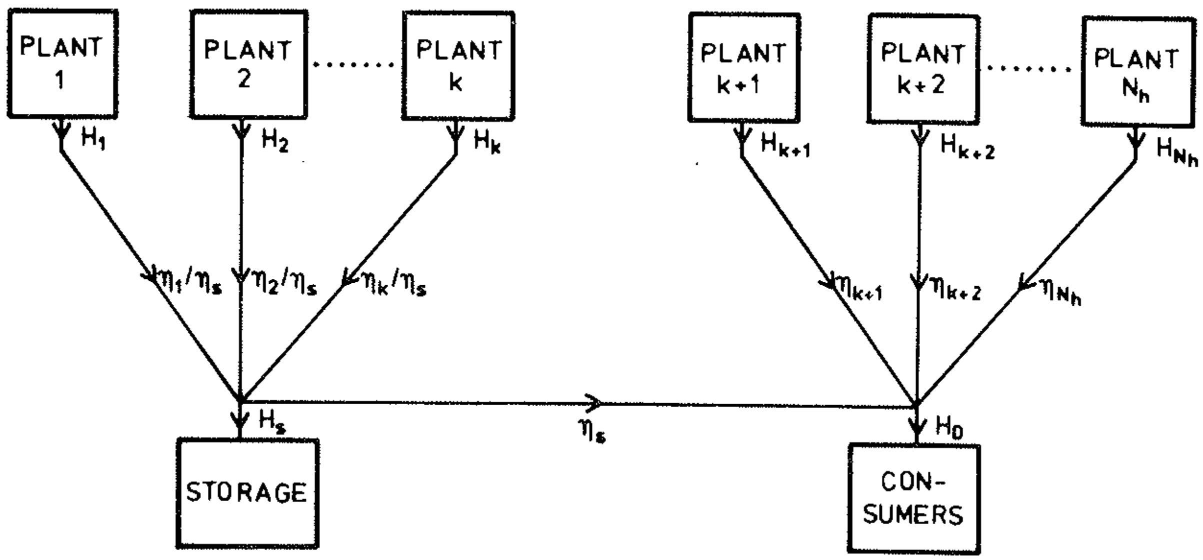

Fig. 5.4. Modelling the system for production, transmission and storage of heat.

The variables used in the mathematical formulation of the problem are as follows:

N_h	:	Number of heat-producing units in the district heating area considered.
i	:	Index for production unit.
N_t	:	Number of time steps.
t	:	Index for time step.
Δt	:	Length of time step (h).
E_{St}	:	Heat contents of the storage facility at the start of time step t (MWh).
H_{St}	:	Heat delivered to the storage facility in time step t (MW). If H_{St} is negative, heat is supplied from the storage.
H_{Lt}	:	Heat losses from storage in time step t (MW).
H_{it}	:	Heat production at unit i in time step t (MW).
H_{Dt}	:	Heat demanded by the consumers in time step t (MW).
H'_{Dt}	:	Apparent heat demand in time step t (MW). $H'_{Dt} = H_{Dt} + \eta_S \cdot H_{St}$
E_S^{max}	:	Maximum energy contents in storage facility (MWh).
E_S^{min}	:	Minimum energy contents in storage facility (MWh). $E_S^{min} = 0$.
H_S^{max}	:	Maximum heat delivered to storage facility (MW).

H_S^{min}	:	$-H_S^{min}$ is the maximum heat supplied from storage unit (MW).
H_i^{max}	:	Maximum heat production at unit i (MW).
H_i^{min}	:	Minimum heat production at unit i (MW). $H_i^{min} = 0$.
η_i	:	Effectivity (1-losses) in transmission of heat from unit i to the consumers (p.u.).
η_S	:	Effectivity (1-losses) in transmission of heat from storage facility to the consumers (p.u.).
C_S	:	Operating costs of storage facility (pumping costs etc.) (kr/MWh in/out).
$C_{tot,t}(H'_{Dt})$	:	Production costs (fuel, O&M*)) in time step t (kr/h). $C_{tot,t}$ is a function of H'_{Dt}.
$C_{marg,t}(H'_{Dt})$	:	Marginal production costs in time step t (kr/MWh). $C_{marg,t} = dC_{tot,t}/dH'_{Dt}$.

The index t on $C_{tot,t}$ and $C_{marg,t}$ indicates that the shape of the functions depends on the time step t. There are two reasons for this: First the price of the co-produced power varies from one time step to another. Secondly a plant might be excluded form production in some time step because it was scheduled for production below technical minimum in an earlier optimization of the operation of the storage facility.

E_{St}, $t=1,2,\ldots,N_t$ are chosen as optimization variables. Therefore the problem of finding the optimal operation schedule for the storage facility can be formulated in the following way:

Minimize

$$f(E_{S1},E_{S2},\ldots,E_{SN_t}) = \Delta t \cdot \sum_{t=1}^{N_t} (C_{tot,t}(H'_{Dt}) + C_S \cdot |H_{St}|)$$

subject to the following inequality constraints:

*) O&M: Operation and Maintenance.

1) $E_S^{min} \leq E_{St} \leq E_S^{max}$ (energy content in storage facility)

2) $H_S^{min} \leq H_{St} \leq H_S^{max}$ (heat into storage facility)

3) $\sum_{i=1}^{N_h} \eta_i \cdot H_i^{min} \leq H'_{Dt} \leq \sum_{i=1}^{N_h} \eta_i \cdot H_i^{max}$ (heat demand)

H_{St} and H'_{Dt} can be expressed as functions of E_{St}, $t=1,2,\ldots,N_t$:

$$E_{S(t+1)} = E_{St} + (H_{St} - H_{Lt}) \cdot \Delta t$$

where $H_{Lt} = c_1 \cdot 0.5 \cdot (E_{St} + E_{S(t+1)}) + c$.

c_1 and c give the heat loss from the storage facility. By inserting this expression for H_{Lt} one gets:

$$H_{St} = a \cdot E_{S(t+1)} - b \cdot E_{St} + c$$

where $a = (1 + 0.5 \cdot c_1 \cdot \Delta t) / \Delta t$
and $b = (1 - 0.5 \cdot c_1 \cdot \Delta t) / \Delta t$.

Likewise for H'_{Dt}:

$$H'_{Dt} = H_{Dt} + \eta_S \cdot H_{St} \qquad (= \sum_{i=1}^{N_h} \eta_i H_{it})$$

$$= H_{Dt} + \eta_S \cdot (a \cdot E_{S(t+1)} - b \cdot E_{St} + c) \quad .$$

It should be borne in mind that the time period is considered cyclic, so

$$E_{S(N_t+1)} = E_{S1} \quad .$$

The above expressions for H_{St} and H'_{Dt} show that these variables are given as linear functions of the energy contents of the storage facility. Therefore for each time step there are 6 linear inequality constraints on the optimization variables. The object function $f(E_{S1}, E_{S2}, \ldots, E_{SNt})$ is nonlinear.

The plants $k+1$, $k+2, \ldots, N_h$ (see Fig. 5.4) that deliver their production directly to the consumers should not be used to charge the storage unit. However, this constraint is not introduced in the present model. But since these plants are district heating boilers while plants 1,2,...,k are CHP plants, the storage facility would not possibly be charged by plants k+1, k+2,...,N_h in the optimal solution in any time steps.

5.4.3. Solution of the problem

The optimization of the operation of the heat storage unit is shown in Fig. 5.5. Three library routines have been tested to solve the linearly constrained nonlinear minimization problem. These routines are: MINLA1 (ref. 24 and 27), MMLA1Q (ref. 25 and 28), and VE01A (ref. 26).

First-order derivatives of the object function is used by all three routines. MMLA1Q and MINLA1 solve the nonlinear minimization problem as a series of linear minimization problems. Moreover, MMLA1Q has the ability to switch to another method, a quasi-Newton algorithm, using an approximation to the inverse Hessian matrix*) of the object function. VE01A also uses a quasi-Newton algorithm, but it does not (like MMLA1Q) contain an algorith for linear optimization.

All three routines need rather many iteration steps to find the optimum. There are two reasons for this: First, the size of the problem is rather large. If, for instance, the simulations cover a period of four days and the time step is two hours, there are 48 optimization variables (E_{St}, t=1,2,...,48) and 288 linear inequality constraints.

Secondly, the shape of the cost surface makes it difficult to reach the optimum. For instance, when a coal-fired CHP plant is operated together with an oil-fired district heating boiler, the

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*) The Hessian matrix H of a function $f(x_1,x_2 \ldots,x_n)$ is defined as the matrix of second-order derivatives:
$H_{ij} = \partial^2 f/\partial x_i \partial x_j$.

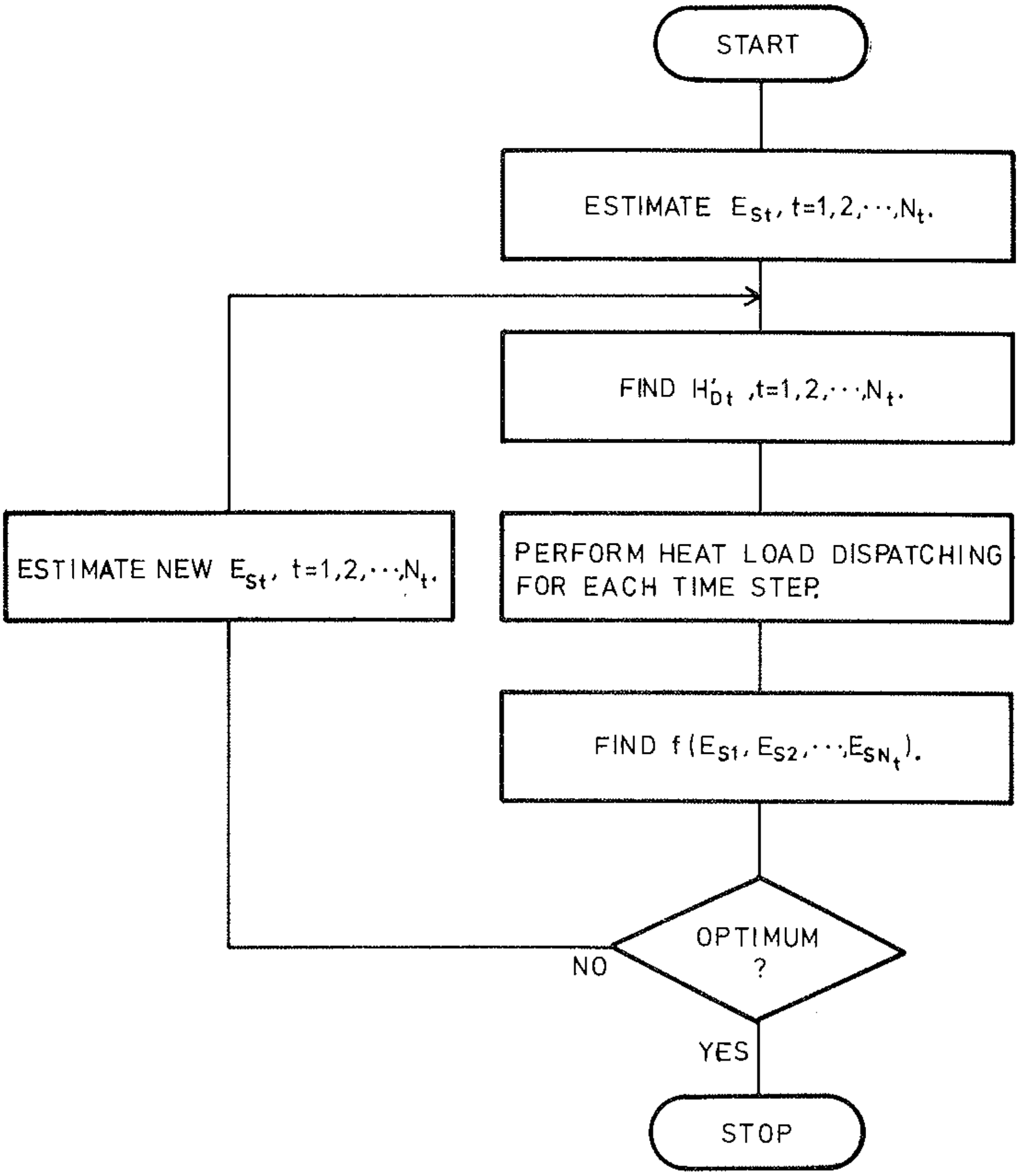
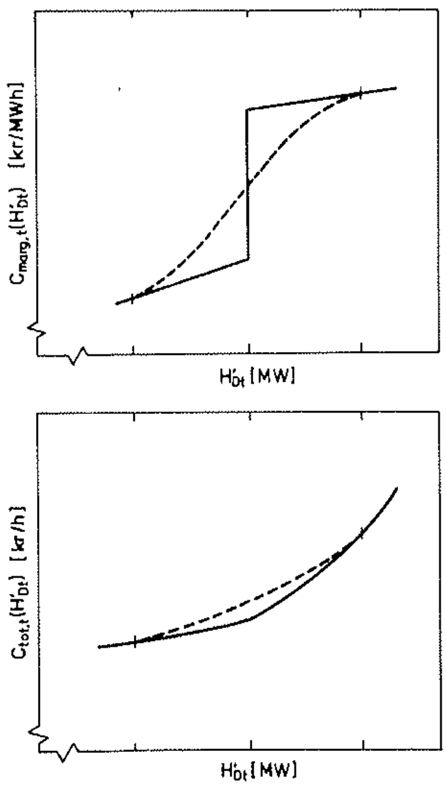


Fig. 5.5. Optimization of the operation of a heat storage unit.

marginal production costs $C_{marg,t}(H'_{Dt})$ will increase very much if the heat demand increases so that the district heating boiler has to take part in the production, too. This discontinuity in $C_{marg,t}$ is removed by smoothing the cost curve $C_{tot,t}(H'_{Dt})$, as shown in Fig. 5.6. But in spite of this smoothing there still exist rather narrow valleys in the cost surface, and moreover the slope of the sides of the valleys is in many cases much greater than the slope of the bottom of the valleys in the direction of the optimum. All three library routines sometimes fail since they reduce the step length because of the difference between the cost surface and the linear approximation to this.

At present, the above-mentioned routines give the correct solution, but the rate of convergence seems too slow. Therefore, this problem should be studied further in the future in order to find a new optimization routine that could increase the computational speed.



<u>Fig. 5.6.</u> Smoothing of discontinuity in $C_{marg,t}$.
Solid line : Correct costs.
Dashed line: Costs as used in the optimizations.

6. EXAMPLES OF SIMULATION

6.1. Introduction

In this chapter the results of some runs of the Simulachron model will be documented. To show the characteristics of the model some small examples will be discussed first. In these examples there is only one district heating area and a very limited number of power plants. In some examples a heat storage facility is also incorporated in the simulation. The time period simulated is very short, only one day divided into 12 time steps of 2 hours.

To show the capability of the model the operation of a total power system consisting of some 30 plants is simulated over 48 time steps. The system dealt with is the ELSAM power system which covers the western half of Denmark (yearly power demand: some 13 TWh). The system includes seven district heating areas with CHP production. There are heat storage facilities in two of these areas. The results of these simulations performed by the Simulachron model are compared with those obtained from the SIM model normally used by the ELSAM power company.

6.2. Small examples

6.2.1. Heat demand satisfied by district heating boiler and by back-pressure power plant

In this example only the heat production is considered. The co-produced electricity from the back-pressure power plant is assumed to be sold to the power system at a given time-dependent price. Moreover this power production is assumed to be so small that it does not change this marginal price of power production for the surrounding power system, the operation of which is not simulated.

Four different ways of satisfying the heat demand have been investigated:

Case 1: The heat production takes place in an oil-fired district heating boiler. There is no CHP plant.

Case 2: A coal-fired back-pressure power plant is added to case 1.

Case 3: A heat storage facility is added to case 2.

Case 4: The back-pressure power plant is not allowed to be operated from midnight to six o'clock in the morning. To make it unnecessary to start the district heating boiler during the night the size of the storage facility is increased.

Although the Simulachron model is mainly intended to be used to simulate the operation of CHP systems it is also usable in case 1.

It has not been attempted to find optimal capacities of the plants and storage facilities. Therefore, the same CHP plant is used in cases 2, 3, and 4 although its size might not be optimal from the point of view of minimizing costs.

The time simulated is one day starting at midnight. The day is divided into twelve time steps of two hours. In the simulation of the operation of the storage facilities the time period of one day is assumed to be cyclic, i.e. the twelfth time step is assumed to be followed by the first one.

The total heat demand is 2930 MWh. In Table 6.1, the demanded heat power is given for each time step together with the price at which the co-produced electricity is sold. This price is assumed to be independent of the power production in the back-pressure plant. In time step 5 the marginal power production in the surrounding power system is assumed to take place at an oil-fired power plant, whereas the available coal-fired plants are able to satisfy the demand in all other time steps. Hence, the selling price of electricity has its maximum value in time step 5.

Table 6.1. Heat demand and price at which the co-produced electricity is sold.

Time step	1	2	3	4	5	6	7	8	9	10	11	12
Heat demand MW	100	105	110	120	125	130	140	150	135	130	115	105
Selling price of electricity kr/MWh	120	125	125	140	300	150	150	145	145	140	135	130

Data describing the back-pressure power plant and the district heating boiler are given in Tables 6.2 and 6.3, respectively. Several district heating boilers with a total capacity of 200 MW are modelled as one 200 MW boiler. The specifications for the heat storage facilities used in cases 3 and 4 are shown in Table 6.4.

The fuel prices are 28 kr/GJ for fuel oil and 13 kr/GJ for coal. The cost of labour is 45 kr/LH (LH = labour hour).

The results of the simulations of cases 1, 2, 3, and 4 will be given below. Figure 6.1 shows for all four cases how the heat production varies during the day. Also shown is the selling price of the co-produced electricity. In Fig. 6.2 and 6.3 the operation of the system is shown for cases 3 and 4.

It is seen that in cases 1 and 2 the production follows the heat demand (there is no storage facility). The difference in production between cases 1 and 2 is due to the difference in

Table 6.2. Data describing the back-pressure power plant.

Minimum power production	30 MW
Maximum power production	100 MW
Maximum heat production	200 MW
Back-pressure constant (see Section 2.3)	0.5 MW/MW
Transmission losses, power	7 %
Transmission losses, heat	15 %
Fuel	Coal
Fuel consumption at minimum production	480 GJ/h
Piecewise linear incremental fuel consumption curve (see Fig. 2.3, method 3):	
Power production	30 MW
	50 MW
	70 MW
	100 MW
Incremental fuel consumption	9.0 GJ/MWh
	10.0 GJ/MWh
	10.5 GJ/MWh
	11.0 GJ/MWh
Variable production costs (excl. fuel)	0.22 LH/MWh
and	0 LH/h
Fixed production costs (excl. fuel)	2.2 LH/kWy
Start-up fuel consumption (see Fig. 2.5):	
"Cold start-up"	960 GJ/start
Time constant for the cooling process	12 h
Fixed start-up costs (excl. fuel)	160 LH/start

transmission losses for the district heating boiler and the back-pressure power plant. In cases 3 and 4 the storage facility is operated so that the income from selling the co-produced electricity is maximized: The production is moved to time steps with a high selling price. More precisely, the production is within the limits of the storage facility and back-pressure plant shared among the time steps so that the marginal cost of heat production (including the income from selling the electri-

Table 6.3. Data describing the district heating boiler.

Minimum production	0 MW
Maximum production	200 MW
Transmission losses	10 %
Losses from boiler	15 %
Fuel	Fuel oil
Variable O&M costs	5 kr/MWh
Fixed O&M costs	4 kr/kWy

Table 6.4. Data describing the heat storage facilities.

	Case 3	Case 4
Maximum energy content	300 MWh	750 MWh
Maximum heat power to storage	50 MW	130 MW
Maximum heat power from storage	-50 MW	-130 MW
Transmission losses (from storage to consumer)	15 %	15 %
Heat losses from storage facility:		
proportional to content	0.0001 MW/MWh	0.0001 MW/MWh
constant losses	0 MW	0 MW
Operating costs	0 kr/MWh	0 kr/MWh

city) is equal for all time steps. For instance, in case 3 the marginal cost of production equals 6.1 kr/MWh through time steps 11, 12, 1, 2, and 3 (time step 1 follows time step 12). Through time steps 4 to 10 the production is limited by the maximum energy content of the heat storage unit, the maximum power fed into the storage, and the rated capacity of the back-pressure power plant.

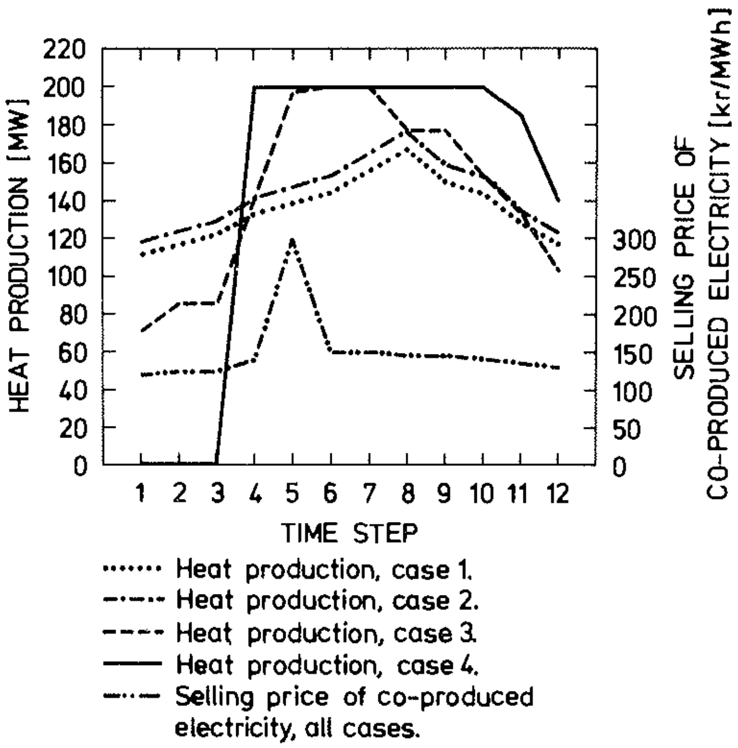


Fig. 6.1. Heat production and selling price of co-produced electricity.

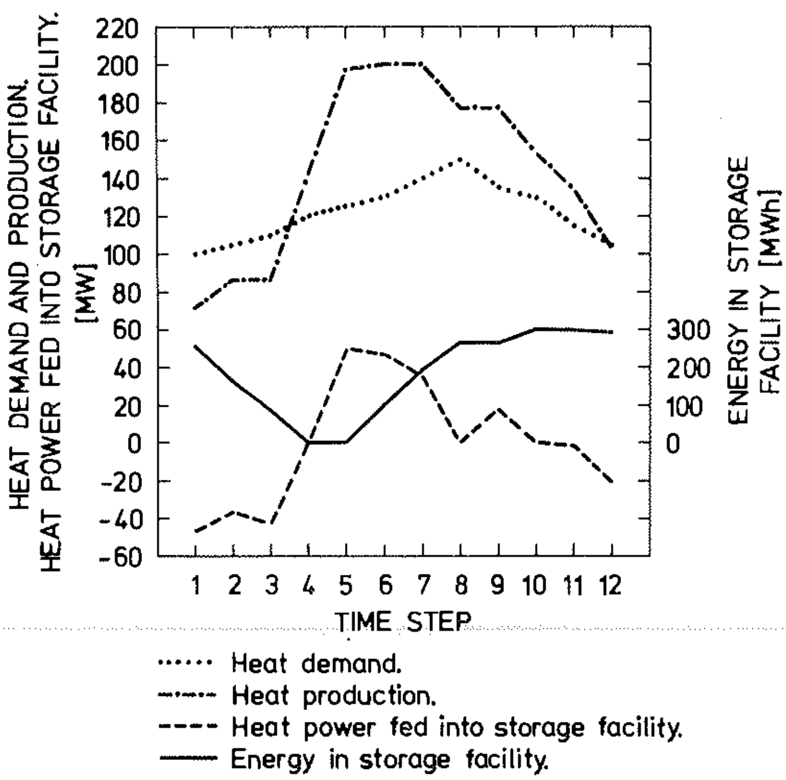


Fig. 6.2. Heat demand and production, and operation of heat storage facility. Case 3.

Also in cases 3 and 4 the oil-fired district heating boiler takes part in the heat-load dispatching. But because of its high marginal production costs it is never operated, the maximum heat power produced being equal to the rated capacity of the back-pressure power plant (200 MW heat).

The costs for the 4 cases are given in Table 6.5. For case 1 it is also shown what the total costs would have been if the district heating boiler had been fuelled by coal at the same price as that used in the back-pressure power plant (13 kr/GJ). Variable and fixed O&M costs (see Table 6.2) are increased when using coal as fuel: 9 kr/MWh and 6 kr/kWy, respectively.

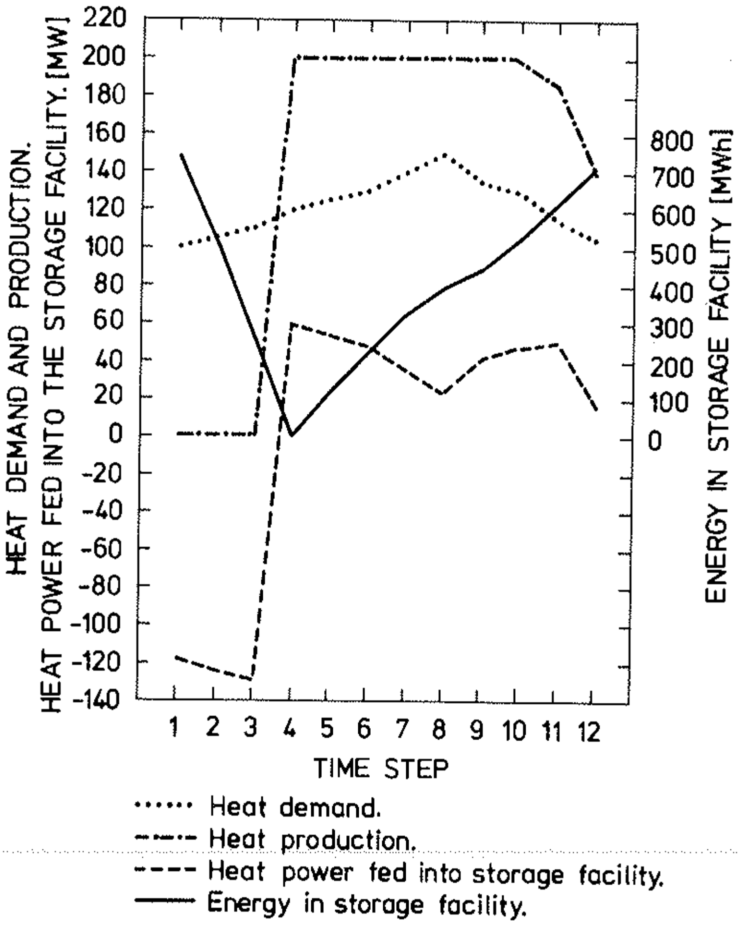


Fig. 6.3. Heat demand and production, and operation of heat storage facility. Case 4.

By comparing cases 1 and 2 it is seen that approximately half of the large economic savings brought about by the coal-fired CHP plant stems from the fuel substitution, while the rest of the savings stems from the co-production. A comparison of cases 2, 3, and 4 shows that the storage facilities give rise to very limited savings (in this example).

Table 6.5. Total costs.

kkr	Case 1	Case 2	Case 3	Case 4
Fuel costs*)	386	279	281	270
O&M costs*)	18	44	44	44
Start-up costs	0	0	0	12
Total production costs	404	323	325	326
Income from selling electricity	0	262	273	277
Total costs	404	61	52	49
Total costs if coal-fired	212			

6.2.2. Heat and power demands satisfied by separate production and by CHP production

In this example both heat and power demands are considered. Three power-producing units are introduced: A base-load plant,

*) Excl. start-up costs.

a medium-load plant, and a peak-load plant. There is only one district heating area supplied from a district heating boiler (case 1), or the medium-load power plant is converted into an extraction power plant producing both heat and power (cases 2 and 3). The introduction of a heat storage facility is simulated, too (case 3).

The three cases simulated by means of the Simulachron model are:

Case 1: Electricity is produced by three power plants, a base-load, a medium-load, and a peak-load plant. Heat is produced by a district heating boiler fired by fuel oil.

Case 2: The medium-load condensing power plant is converted into an extraction power plant. The district heating boiler is still present.

Case 3: The system is equal to the one described for case 2, but a heat storage facility is added.

As was the case in the previous example the simulations cover a period of one day starting at midnight. The day is divided into twelve time steps of two hours.

In Table 6.6 the heat and power demands are given. The heat demand is the same as used in the previous example (see Table 6.1). The district heating boiler is specified in Table 6.3, and data describing the storage facility used in case 3 are given in Table 6.7. The three power plants are specified in Table 6.8. The medium-load plant is described as an extraction power plant in the table, but in case 1 it is operated as a condensing power plant, i.e. it is not allowed to produce heat.

The fuel prices are 28 kr/GJ for fuel oil and 13 kr/GJ for coal. The cost of labour is 45 kr/LH (LH = labour hour).

In the simulation of the unit commitment a total optimization has been performed, i.e. no sub-optimizations are carried out (see Section 4.4). None of the optional constraints are used (see Section 4.2.2). The simulations in this example are performed with no requirement of spinning reserve (constraint number 1). Although the model allows for a specification of the power transmission losses by B-constants (see Section 2.8) the losses are given as a constant fraction (7%) of the power production.

Table 6.6. Heat and power demands.

Time step		1	2	3	4	5	6	7	8	9	10	11	12
Heat demand	MW	100	105	110	120	125	130	140	150	135	130	115	105
Power demand	MW	160	170	180	280	350	330	320	320	310	290	250	170

Table 6.7. Data describing the heat storage facility.

Maximum energy content	100	MWh
Maximum heat power to storage	33.8	MW
Maximum heat power from storage	-33.8	MW
Transmission losses (from storage to consumer)	15	%
Heat losses from storage facility:		
proportional to content	0.0001	MW/MWh
constant losses	0	MW
Operating costs	0	kr/MWh

The results of the simulations are given in Figs. 6.4 to 6.8 and in Tables 6.9, 6.10, and 6.11. By comparing cases 1 and 2 in Figs. 6.4, 6.5, and 6.7 it is seen that the heat production at the medium-load extraction plant reduces its maximum power production and that the peak-load plant therefore has to

Table 6.8. Data describing the three power plants.

	Base-load plant	Medium-load plants	Peak-load plants	Units
Minimum power production	55	35	13	MW
Maximum power production	270	125	60	MW
Maximum heat production	0	160	0	MW
Back-pressure constant		0.51		MW/MW
c_V-constant		0.15		MW/MW
Transmission losses, power		7		%
Transmission losses, heat		15		%
Fuel	Coal	Coal	Fuel oil	
Fuel consumption at minimum production	560	430	290	GJ/h
Piecewise linear incremental fuel consumption curve:				
Power production	55	35	13	MW
	80	50	20	MW
	150	90	50	MW
	230	120	60	MW
	250	125		MW
	270			MW
Incremental fuel consumption	7.5	9.0	10.0	GJ/MWh
	7.8	9.3	11.0	GJ/MWh
	8.6	9.7	13.0	GJ/MWh
	9.5	12.0	30.0	GJ/MWh
	9.8	12.6		GJ/MWh
	10.1			GJ/MWh
Variable O&M costs	0.17	0.20	0.30	LH/MWh
and	0	0	0	LH/h
Fixed O&M costs	0.4	0.5	0.8	LH/kWy
Start-up fuel consumption:				
"Cold start-up"	1200	400	300	GJ/start
Time constant of cooling process	12	8	6	h
Fixed start-up costs	630	200	80	LH/start

be started in time step 5 where the power demand is at its maximum. Moreover, by comparing cases 2 and 3 it is seen (Fig. 6.8) that the heat storage facility in case 3 is operated so that the heat production in time step 5 is reduced just enough to make it unnecessary to start the peak-load plant.

The maximum heat production by the CHP plant is 160 MW. Therefore, in case 2 the district heating boiler takes part in the production during time steps 7 and 8 (Fig. 6.6). In case 3, however, the storage facility is operated so that heat production is moved away from these time steps. Therefore, the CHP plant supplies all the heat.

The discussion above shows that the storage facility has two main objectives: To reduce the expensive production of heat and power by the two peak-load units (the district heating boiler

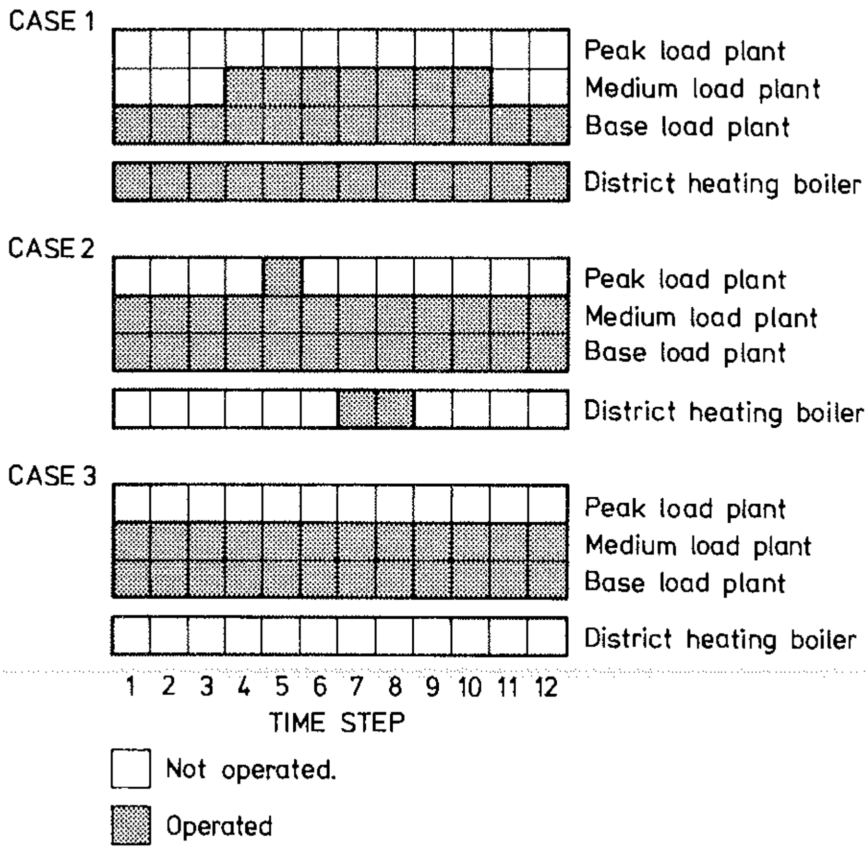


Fig. 6.4. Operation of power plants and district heating boiler.

and the peak-load power plant). Tables 6.10 and 6.11 show that the production costs at the district heating boiler is reduced by 5 - 0 = 5 kkr, and at the peak-load power plant by 34 - 6 = 28 kkr. At the same time the production costs at the base-load plant is reduced by 624 - 619 = 5 kkr, and at the medium-load plant increased by 355 - 343 = 12 kkr. Thus the net reduction is 5 + 28 + 5 - 12 = 26 kkr.

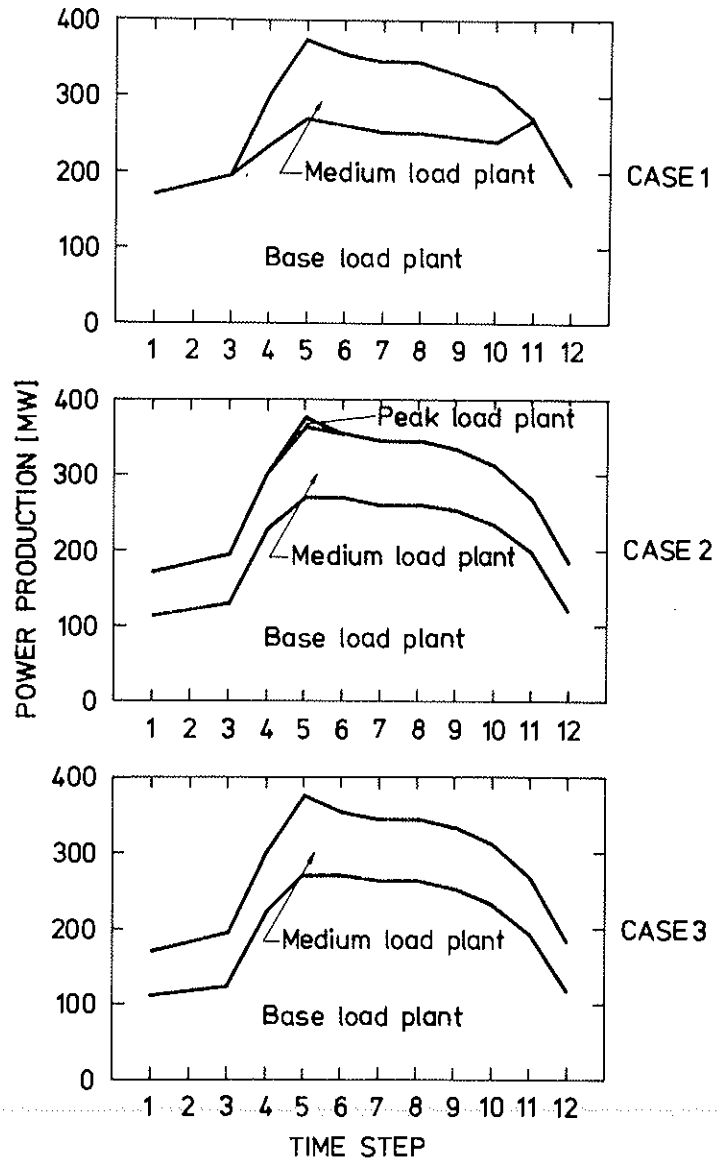


Fig. 6.5. Power production.

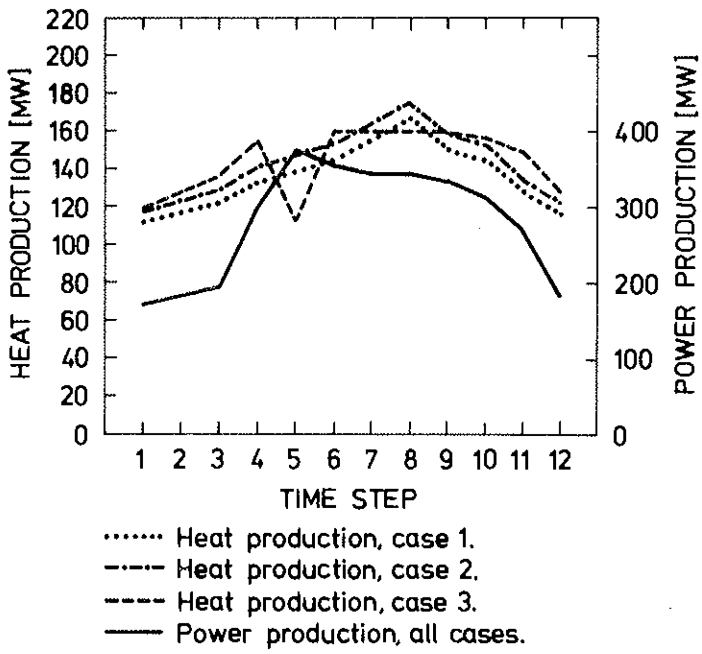


Fig. 6.6. Heat and power production.

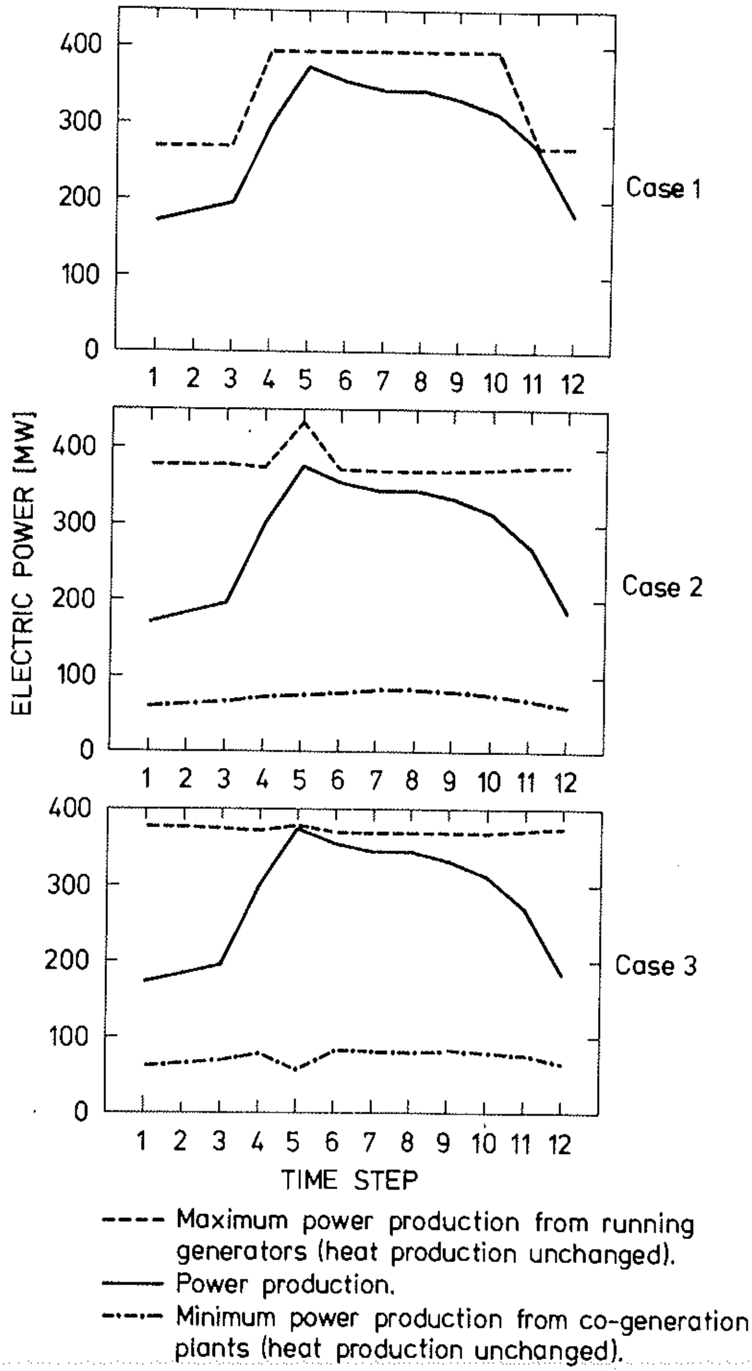


Fig. 6.7. Limits on power production set by heat production. In case 1 there is no CHP production.

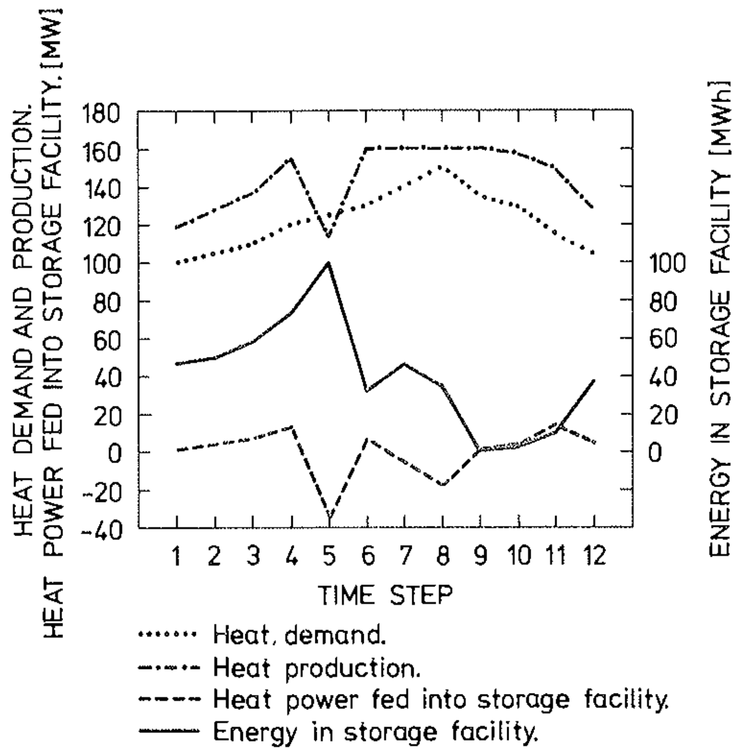


Fig. 6.8. Heat demand and production, and operation of heat storage facility. Case 3.

Table 6.9 Production costs. Case 1.

kkr	Base-load plant	Medium-load plant	Peak-load plant	District heating boiler	Total
Fuel costs*)	642	167	0	386	1195
O&M costs*)	56	19	6	33	114
Start-up costs	0	13	0	0	13
Total	698	199	6	419	1322

*) Excl. start-up costs.

Table 6.10. Production costs. Case 2.

kkr	Base-load plant	Medium-load plant	Peak-load plant	District heating boiler	Total
Fuel costs*)	573	315	16	5	909
O&M costs*)	51	28	6	0	85
Start-up costs	0	0	12	0	12
Total	624	343	34	5	1006

Table 6.11. Production costs. Case 3.

kkr	Base-load plant	Medium-load plant	Peak-load plant	District heating boilers	Total
Fuel costs*)	568	326	0	0	894
O&M costs*)	51	29	6	0	86
Start-up costs	0	0	0	0	0
Total	619	355	6	0	980

*) Excl. start-up costs.

6.3. Full-size example

6.3.1. The CHP system simulated

The CHP system that is dealt with here is the total ELSAM power system which covers the western half of Denmark. In 1982 the power demand was some 13 TWh (ref. 13 and 16). The system includes seven district heating areas supplied with heat from CHP plants. In the same year the heat demand in these areas was some 8 TWh (ref. 13).

The total installed capacity of the 31 available power plants is 3642 MW (1983). Among these power plants there are 15 extraction power plants, two back-pressure power plants, and one gas turbine. The ELSAM power system is connected to the Norwegian, Sweedish, and German power systems by transmission lines of several hundred megawatts. These inter-system connections are not covered by the present version of the Simulachron model.

Due to the detailed modelling in Simulachron the data requirements for the ELSAM CHP production system are heavy. Therefore all data will not be listed as was the case for the small examples. Instead the data can be found in ref. 13.

Peak-power district heating boilers fuelled by oil are modelled for each district heating area. Also included are two refuse incineration plants that are operated as base-load heat production units the production of which is limited by the amount of refuse available - and of course by the technical capacity of the plants.

Most power plants are fuelled by coal at 18.85-20.65 kr/GJ, while others are fuelled by oil at 38.00-38.20 kr/GJ. These intervals reflect different costs of transporting fuel to the plants. Oil for the gas turbine costs 59.00 kr/GJ. District heating boilers are fuelled by oil at 40.00 GJ.

Two day-to-day heat storage facilities are included in the simulations. One is operated in connection with three extraction power plants (in Odense), and the other one in connection with a back-pressure power plant (in Herning). According to ref. 13 the storage unit in Odense can hold 695 MWh of heat. It can be charged by 100 MW and de-charged by 280 MW. Because different forward temperatures are used in the district heating system in Herning the maximum ratings of the storage unit differ from summer to winter. The maximum energy content is 1048 MWh (1223 MWh). The maximum stored power is 135 MW (157 MW) and the maximum power extracted from the storage unit is 139 MW (162 MW). The first figures are valid during the summer, and those in parentheses during the winter.

The operation of the ELSAM CHP production system has been simulated for a winter and summer week. To keep down the computer time only 4 days a week are simulated: Saturday, Sunday, Monday, and an average "normal working day". This last type of day represents Tuesday to Friday. Fuel consumption, costs, etc., found for this day are multiplied by four to give the correct results. With a time step of length two hours the simulations cover 4•24/2=48 time steps.

Not all 31 power plants take part in the simulations: In the winter week one peak-load plant is unavailable due to planned maintenance, and in the summer week seven plants are unavailable. (The maintenance planning is outside the scope of this study).

The heat and power demands valid for 1983 are given in Table 6.12. In the winter the heat demand is made up mainly by the need for residential heating, but also hot tap water and transmission losses (see below) are included. In the summer very little energy is demanded for residential heating whereas the need for hot tap water and the losses depend very little on the time of the year. The heat load profiles in Fig. 6.9 are common to all district heating areas. Figure 6.10 shows the power demand. (All figures referenced in Section 6.3 are placed at the end of the section.)

The demands are as given in refs. 13 and 23 for period 4 (winter) and period 14 (summer). Reference 23 gives some information on how

Table 6.12. Heat and power demand (7 days).

District heating area	Winter week GWh	Summer week GWh
Heat demand:		
Odense	97.0	16.8
Herning	29.8	5.2
Århus	49.9	8.7
Ålborg	50.3	8.7
Randers	21.5	3.7
Åbenrå	8.1	1.4
Esbjerg	50.3	8.7
Total heat demand	306.8	53.3
Power demand	286.3	206.8

these standard load curves are found from measurements of real loads. Both heat and power demands should be interpreted as loads measured at the power plants, and not as heat and power demanded by the consumers. Therefore transmission losses are not considered explicitly in the simulations. As a consequence of this all incremental power transmission losses used in the load dispatching procedure equal zero (see Section 4.3.4).

6.3.2. Results of simulations

In this section the results of the simulations will be shown and commented. Because of the amount of output from the simulations only overall results will be given in tables while more detailed results will be shown graphically.

Table 6.13 and 6.14 give information on the production at the power plants for the winter and summer weeks considered. The costs associated with these productions are shown in Tables 6.15 and 6.16.

The total production costs (CHP plants, condensing plants, and district heating boilers) cannot be divided unambigously between the heat and power productions to give costs per produced MWh of heat and per MWh of power. Instead, the connection between heat and power prices must be given as in Fig. 6.11: If it is decided to sell the electricity at e.g. 200 kr/MWh, the corresponding price at which the heat should be sold to balance the total production costs (reduced by the income from selling the electricity) is found where a horizontal line through 200 kr/MWh intersects the appropriate line (summer or winter) in the diagram. It should be noted that no depreciation of the investments is included. Also given in Fig. 6.11 are the specific production costs for a modern condensing power plant and for a district heating boiler. The specific costs are shown for oil-firing and coalfiring. Moreover it is shown what the specific cost of power production for the ELSAM power system would be if no heat was produced. Both for the winter and summer weeks considered it is seen that economic savings are brought above by the co-production of heat and power.

The operation of each power plant is shown in Figs. 6.12 and 6.13 for the first four days of the winter and summer weeks, respectively. The plants are sorted according to the ratio of produced electric energy to the maximum actually produced power. An identification of the plants is given in the lower part of the figures. It is seen that some plants are operated as base-load plants producing on almost constant amount of electricity while others are operated as medium-load plants that are taken out of operation during night hours. Peak-load plants are operated for only a few hours a day during one or both of the two daily load peaks. Some of the plants operated at an almost constant level of power production are not base-load plants - from a power production point of view. Instead their mode of operation stems from the heat production scheduling - and as described in Chapter 2 CHP plants usually cannot produce heat without generating a certain amount of power, too.

Figures 6.14 and 6.15 give some information on the operation of the total power production system. The uppermost curve gives the rated capacity of the running plants. The second uppermost and the second lowest curves give the interval within which the

Table 6.13. Heat and power production. Winter week.

Plant	Type of plant *)	Production of electricity GWh**)	Production of heat GWh**)	Fuel consumption incl. start-up TJ**)	Capacity factor ***)	Number of start-ups
FVO 3	extr.	6.2	12.1	98.6	0.67	6
FVO 4	extr.	23.2	39.1	281.9	0.86	0
FVO 6	extr.	34.5	45.8	395.2	0.94	0
MKA 8	extr.	2.2	8.7	47.7	1.00	0
MKA 9	extr.	7.2	15.6	106.5	0.79	0
MKA 10	extr.	7.2	15.6	106.5	0.79	0
MKS 1	cond.	16.2		150.8	0.64	0
MKS 2	cond.	0.		0.	0.	0
NEV 1	cond.	9.7		97.3	0.44	4
NEV 2	cond.	0.		0.	0.	0
NEV 3	gas turb.	0.		0.	0.	0
NKA 3	cond.	0.		0.	0.	0
NKA 4	cond.	0.		0.	0.	0
NKA 5	extr.	4.1	8.6	67.6	0.78	0
NKA 6	extr.	7.3	16.9	120.6	0.82	0
NKA 8	extr.	13.3	21.6	153.3	0.36	0
SVS 1	cond.	0.		0.	0.	0
SVS 2	cond.	0.		0.	0.	0
SVS 3	cond.	0.		0.	0.	0
SVS 21	cond.	31.6		285.8	0.70	0
SHA 5	cond.	0.		0.	0.	0
SHE 6	cond.	1.3		19.0	0.13	5
SHE 7	extr.	14.1	4.9	150.9	0.61	0
SHE 8	extr.	43.2	3.2	386.5	0.87	0
VKE 4	extr.	0.2	0.7	9.8	0.06	9
VKE 5	extr.	1.7	3.6	29.0	0.22	5
VKE 6	extr.	12.5	13.2	151.8	0.69	0
VKE 7	extr.	28.4	32.7	309.4	0.81	0
RMO 1	b.-p.	7.6	17.6	104.6	1.00	0
HMO 1	b.-p.	14.6	29.2	181.7	1.00	0
Total, plants		286.3	289.3	3254.4		
Total, boilers****)			17.6	90.3		
Total, plants and boilers****)		286.3	306.9	3344.8		
Loss, storage units			0.0			
Demand		286.3	306.9			

*) cond. = condensing. extr. = extraction. b.-p. = back-pressure.

**) Exact zero is written as 0. while a positive value less than 0.05 is written as 0.0.

***) Capacity factor is the ratio of energy generated by a unit over a period of time to the energy which would have been generated had it operated continuously at its maximum capability during the period. Exact zero is written as 0. while a positive value less than 0.005 is written as 0.00.

****) Incl. refuse incineration plants.

Table 6.14. Heat and power production. Summer week.

Plant	Type of plant *)	Production of electricity GWh**)	Production of heat GWh**)	Fuel consumption incl.start-up TJ**)	Capacity factor ***)	Number of start-ups
FVO 4	extr.	24.8	16.8	271.5	0.82	0
MKA 8	extr.	1.5	5.4	34.7	0.66	0
MKA 9	extr.	5.6	0.0	66.1	0.48	1
MKA 10	extr.	6.7	1.0	80.7	0.58	0
MKS 1	cond.	22.9		209.2	0.90	0
MKS 2	cond.	0.		0.	0.	0
NEV 2	cond.	0.		0.	0.	0
NKA 3	cond.	0.4		7.8	0.09	6
NKA 4	cond.	0.3		5.3	0.06	5
NKA 5	extr.	1.5	0.	19.4	0.21	5
NKA 6	extr.	5.0	5.5	76.6	0.49	0
NKA 8	extr.	0.	0.	0.	0.	0
SVS 1	cond.	0.		0.	0.	0
SVS 2	cond.	0.		0.	0.	0
SVS 3	cond.	1.3		19.2	0.13	5
SVS 11	cond.	13.8		133.9	0.81	0
SVS 21	cond.	44.5		397.0	0.98	0
SHA 5	cond.	0.0		0.4	0.00	1
SHE 6	cond.	2.6		34.1	0.22	5
SHE 7	extr.	21.2	1.4	212.3	0.88	0
VKE 4	extr.	0.3	0.	6.1	0.05	5
VKE 6	extr.	14.6	0.	152.1	0.69	1
VKE 7	extr.	37.3	8.7	357.6	0.94	0
HMO 1	b.-p.	2.6	5.2	43.1	0.21	9
Total, plants		206.8	44.2	2127.0		
Total, boilers****)			9.1	54.6		
Total, plants and boilers****)		206.8	53.3	2181.5		
Loss, storage units			0.0			
Demand		206.8	53.3			

*) cond. = condensing. extr. = extraction. b.-p. = back-pressure.

**) Exact zero is written as 0. while a positive value less than 0.05 is written as 0.0.

***) Capacity factor is the ratio of energy generated by a unit over a period of time to the energy which would have been generated had it operated continuously at its maximum capability during the period. Exact zero is written as 0. while a positive value less than 0.005 is written 0.00.

****) Incl. refuse incineration plants.

Table 6.15. Production costs. Winter week.

Plant	Type of plant *)	Fuel costs excl.start-up Mkr	Operating costs Mkr	Start-up costs Mkr	Total costs Mkr
FVO 3	extr.	1.911	0.154	0.085	2.150
FVO 4	extr.	5.596	0.342	0.000	5.938
FVO 6	extr.	7.846	0.481	0.000	8.327
MKA 8	extr.	0.927	0.096	0.000	1.023
MKA 9	extr.	2.072	0.170	0.000	2.242
MKA 10	extr.	2.072	0.170	0.000	2.242
MKS 1	cond.	2.994	0.236	0.000	3.230
MKS 2	cond.	0.000	0.105	0.000	0.105
NEV 1	cond.	1.877	0.166	0.118	2.161
NEV 2	cond.	0.000	0.088	0.000	0.088
NEV 3	gas turb.	0.000	0.030	0.000	0.030
NKA 3	cond.	0.000	0.032	0.000	0.032
NKA 4	cond.	0.000	0.032	0.000	0.032
NKA 5	extr.	1.369	0.117	0.000	1.487
NKA 6	extr.	2.443	0.172	0.000	2.615
NKA 8	extr.	5.857	0.167	0.000	6.024
SVS 1	cond.	0.000	0.034	0.000	0.034
SVS 2	cond.	0.000	0.034	0.000	0.034
SVS 3	cond.	0.000	0.047	0.000	0.047
SVS 21	cond.	5.558	0.386	0.000	5.945
SHA 5	cond.	0.000	0.046	0.000	0.046
SHE 6	cond.	0.317	0.064	0.077	0.458
SHE 7	extr.	2.844	0.226	0.000	3.070
SHE 8	extr.	7.286	0.455	0.000	7.741
VKE 4	extr.	0.154	0.040	0.067	0.262
VKE 5	extr.	0.540	0.076	0.061	0.677
VKE 6	extr.	3.013	0.220	0.000	3.233
VKE 7	extr.	6.141	0.391	0.000	6.532
RMO 1	b.-p.	2.159	0.166	0.000	2.325
HMO 1	b.-p.	3.752	0.358	0.000	4.110
Total, plants		66.727	5.103	0.408	72.239
Total, boilers**)		2.066	0.378		2.444
Total, plants and boilers**)		68.793	5.481	0.408	74.682

*) cond. = condensing. extr. = extraction. b.-p. = back-pressure.
**) Incl. refuse incineration plants.

Table 6.16. Production costs. Summer week.

Plant	Type of plant *)	Fuel costs excl.start-up Mkr	Operating costs Mkr	Start-up costs Mkr	Total costs Mkr
FVO 4	extr.	5.390	0.330	0.000	5.720
MKA 8	extr.	0.674	0.072	0.000	0.746
MKA 9	extr.	1.267	0.122	0.025	1.413
MKA 10	extr.	1.570	0.138	0.000	1.708
MKS 1	cond.	4.153	0.302	0.000	4.455
MKS 2	cond.	0.000	0.105	0.000	0.105
NEV 2	cond.	0.000	0.088	0.000	0.088
NKA 3	cond.	0.137	0.040	0.036	0.213
NKA 4	cond.	0.090	0.037	0.030	0.157
NKA 5	extr.	0.373	0.059	0.038	0.470
NKA 6	extr.	1.550	0.122	0.000	1.673
NKA 8	extr.	0.000	0.107	0.000	0.107
SVS 1	cond.	0.000	0.034	0.000	0.034
SVS 2	cond.	0.000	0.034	0.000	0.034
SVS 3	cond.	0.337	0.067	0.063	0.467
SVS 11	cond.	2.604	0.212	0.000	2.816
SVS 21	cond.	7.722	0.500	0.000	8.223
SHA 5	cond.	0.006	0.046	0.015	0.068
SHE 6	cond.	0.417	0.068	0.071	0.764
SHE 7	extr.	4.003	0.294	0.000	4.297
VKE 4	extr.	0.087	0.039	0.049	0.175
VKE 6	extr.	3.002	0.221	0.033	3.256
VKE 7	extr.	7.098	0.438	0.000	7.536
HMO 1	b.-p.	0.799	0.216	0.161	1.177
Total, plants		41.470	3.711	0.521	45.702
Total, boilers**)		0.634	0.336		0.970
Total, plants and boilers**)		42.105	4.046	0.521	46.672

power production can take place without changing the heat production schedule and without starting or stopping any plants (i.e. without changing the unit commitment schedule). In the simulations a 5% spinning reserve is demanded. Therefore the second uppermost curve always lies 5% above the third curve that gives the total power production (which equals the power demand since transmission losses are included in the demand in these examples).

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*) cond. = condensing. extr. = extraction. b.-p. = back-pressure.
**) Incl. refuse incineration plants.

The curve at the bottom of Figs. 6.14 and 6.15 shows the power that would be produced if only heat producing CHP plants were operated - and if they were operated at the lowest possible level of power production without changing the heat production schedule. If the power demand for some time steps were smaller than this minimum power production some special action had to be taken as described in Sections 1.1 and 3.3. Figure 6.14 gives a warning that this problem in the future might arise during cold winter nights if the heat load at CHP plants should be increased without an equivalent increase of power demand. In Section 1.1 it is argued that according to current plans (ref.15) the demand for co-produced heat by 2000 will have increased by 47% relative to the demand for electricity on that date.

The two district heating areas (Odense and Herning) equipped with day-to-day heat storage facilities will be focussed on next. The production schedule for these two district heating areas and the operation of the two storage units are shown in Figs. 6.16 to 6.23. The price at which the co-produced power (during the simulation of the heat production) is assumed to be sold is plotted, too. This price which is a result of the power production simulation governs the operation of the storage facilities.

In Odense there are three extraction power plants (two of which due to planned maintenance are unavailable during the summer week simulated), and in Herning there is one back-pressure power plant. The storage unit in Herning is operated so that the production is moved to time steps with a high price of power. The operation of the storage unit in Odense is more complicated and can be understood only if reference is made to Chapter 5: During night hours when the price of co-produced power is small the extraction power plants are operated at the back-pressure line (cases 1 and 2 in Section 5.2.3) - and therefore the heat production is moved <u>from</u> time steps with relatively low price of power <u>to</u> time steps with a higher price of power within these night hours. On the other hand, during working hours where the price of power is high the extraction power plants are operated at a level of power production near their rated capacity (cases 3 and 4 in Section 5.2.3). Therefore, to be able to produce as much power as possible during time steps of maximum price of co-produced power, heat production is moved away from these time steps.

In cases 3 and 4 (and partly case 2) the incremental costs λ of heat production at an extraction power plant are constant (see Section 5.3.2). Therefore the cost function is linear within a certain sub-space of the solution space. As a result of this the optimum production schedule might change drastically for only small changes of the price of co-produced power. This is seen in Figs. 6.20 and 6.21 where the heat production is varying in a rather oscillatory manner. This production schedule is optimal in the way that the system is modelled. To damp these oscillations a cost for operating the Odense storage facility has been introduced. It is specified as 10 kr per MWh of heat charged into or discharged from the storage unit. Figures 6.24 and 6.25 show the effect of this operating cost. It is seen that the oscillations are damped although they have not quite disappeared.

To find the value (defined as saved production costs) of the storage units simulations have also been performed without these facilities. By comparing the total cost with and without a particular unit it is found what is gained by the storage unit. This comparison has been carried out in Table 6.17. Since the simulations documented here cover only two weeks of the year it is not possible to find the payback time by comparing the saved production costs with investments.

The value of a storage facility could also have been found without simulating the operation of the total CHP system. Instead a simulation of the CHP production in only the district heating area in question (with and without the storage unit) would suffice. In such simulations one needs a time series specifying the price at which the co-produced power is sold to the surrounding power system. This time series could be found in an initial simulation of the operation of the total system. If this method is used one must assume that the marginal costs of

Table 6.17. The value of storage facilities. (Saved weekly total production costs.)

	Winter week Mkr	Summer week Mkr
Total costs with both storage units	74.68	46.67
Total costs without Odense storage unit	74.96	47.31
Total costs without Herning storage unit	74.82	47.80
Value of Odense storage unit	0.28	0.64
Value of Herning storage unit	0.14	1.13

power production for the total system depend only weakly on the operation of the CHP plants in the district heating area considered.

6.3.3. Convergence of over-all iterations

In Chapter 3 it is described how the simulation of the operation of a CHP production system is split up into a simulation of the heat production followed by a simulation of the power production. These two simulations are linked together by the marginal cost of power production that is found during the simulation of the power production and fed back to the heat production simulation which is then repeated. In this way an iteration around the total simulation is performed. This iteration is stopped when the total costs stabilize at a constant level.

In Fig. 6.26 the total costs for each iteration are plotted for the two examples dealt with in Section 6.3.2. It is seen that an accuracy of 0.1% is obtained within 5-10 iterations. As mentioned in Section 3.4 one or two iterations would suffice if no heat storage facilities were present.

6.3.4. Comparison with the SIM model

To ensure that the production schedule found by the Simulachron model is reasonable (i.e. minimum cost) a comparison with the results from runs of the SIM model has been performed. SIM has been used for several years by the ELSAM power company in the yearly expansion planning. A short description of the SIM model is given in ref. 23.

Regarding the modelling of the demands and the technical components of the CHP production system Simulachron and SIM are rather similar. Therefore, it has been possible to use the data listed in ref. 13 in the Simulachron simulations with very few changes.

As a contrast to this similarity the philosophies used in the simulations in the two models are very different: In SIM the experience gained from the daily operation of the CHP system is to a great extent utilized. This means that the simulations are performed rather quickly, but on the other hand one cannot be sure that the result is optimal since only very few candidate solutions are evaluated and compared with regard to their associated costs.

For instance, the procedure for unit commitment is based on two priority lists: a "peak-load" and a "base-load" priority list. In the "base-load" priority list the plants are sorted according to their minimum marginal cost of production and in the "peak-load" priority list according to start-up costs per MW rated capacity. All plants are present in both priority lists.

In Simulachron, however, only the physical constraints on the system limit the solution space. Therefore, the need for computer power is rather large if the optimum solution is sought (see ε-optimality in Appendix A). In the simulation of the unit commitment it is, however, possible to introduce some optional constraints that from a physical point of view are unnecessary, but from an economical point of view seem reasonable (see Section 4.2.2).

Other differences between the two models shall be given below. First it should be mentioned that the present version of the Simulachron model, unlike the SIM model, is unable to simulate power connections to neighbouring power systems.

Secondly, the connection between power production and heat production is dealt with differently in the two models. In Simulachron a time-dependent marginal cost of power production is used when optimizing the heat production, whereas a constant power cost specific to each CHP plant is used in SIM. One major disadvantage of this is that the operation of heat storage facilities cannot be handled by the SIM model as an integral part of the simulations.

However, in connection with back-pressure power plants (but not extraction power plants) a coarse simulation of the operation of a heat storage unit is performed by demanding that the heat energy required by the consumers throughout each day should be produced during as few consecutive time steps as possible. These hours of production are placed at maximum electricity demand. If the necessary time of production is longer than 16 hours the back-pressure power plant is operated throughout the day.

Thirdly, the SIM model considers the days one at a time while Simulachron links together all days (e.g. 4 days) in the simulated period and assumes that the first day follows the last one.

The examples given in Sections 6.3.1 and 6.3.2 have also been simulated by the SIM model. Since this model is unable to simulate the operation of the storage unit in Odense - and only to a certain degree the storage unit in Herning - simulations without these storage facilities have been carried out by Simulachron, too.

SIM does not - like Simulachron - contain a model for peak-load district heating boilers. Therefore SIM ends up with an unserved demand for heat energy. To be able to compare the results from SIM and Simulachron this heat energy is assumed to be produced at district heating boilers identical to the boilers used in the Simulachron computations - and fired by the same fuel at 40 kr/GJ. Also fixed operating costs for non-operated district heating boilers are included.

Another difference between the SIM and Simulachron simulations is that heat production at refuse incineration plants*) is included in Simulachron but not in SIM. Therefore, the heat production at these plants is assumed to be produced at CHP plants in the same district heating area. If the total rated capacity of the available CHP plants is not large enough oil-fired district heating boilers are used.

The total costs found by SIM and Simulachron are shown in Table 6.18. Because of the above-mentioned alterations for refuse incineration plants the total costs for Simulachron in Table 6.18 are slightly greater than those given in Table 6.17. The differences in costs relative to the results from SIM are given in Table 6.18, too. It is seen that the cost difference without the Odense storage unit and with the Herning unit are less than one per cent. But on the other hand the differences are greater than the accuracy (0.1%) of the total costs found by Simulachron (see Fig. 6.26). The small differences between the results from SIM and Simulachron can be explained by the different ways in which the CHP production is modelled (as discussed above).

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*) The two refuse incineration plants are assumed to be operated at a constant level of production throughout the year producing 13 MW and 19 MW of heat, respectively.

Table 6.18: Total production costs found by the SIM and Simulachron models.

Model	Storage unit present		Winter week		Summer week	
	Odense	Herning	Costs Mkr	Diff. %	Costs Mkr	Diff. %
SIM	no	(yes)	75.68	0.0	47.79	0.0
Simulachron	yes	yes	75.06	-0.8	46.74	-2.2
Simulachron	no	yes	75.33	-0.5	47.36	-0.9
Simulachron	yes	no	75.19	-0.7	47.83	0.1
Simulachron *)	no	no	75.46	-0.3	48.45	1.4

*) Simulations without both storage units have not been performed. Instead it is supposed that the increase in total costs when removing one of the storage units does not depend on whether the other storage unit is present or not.

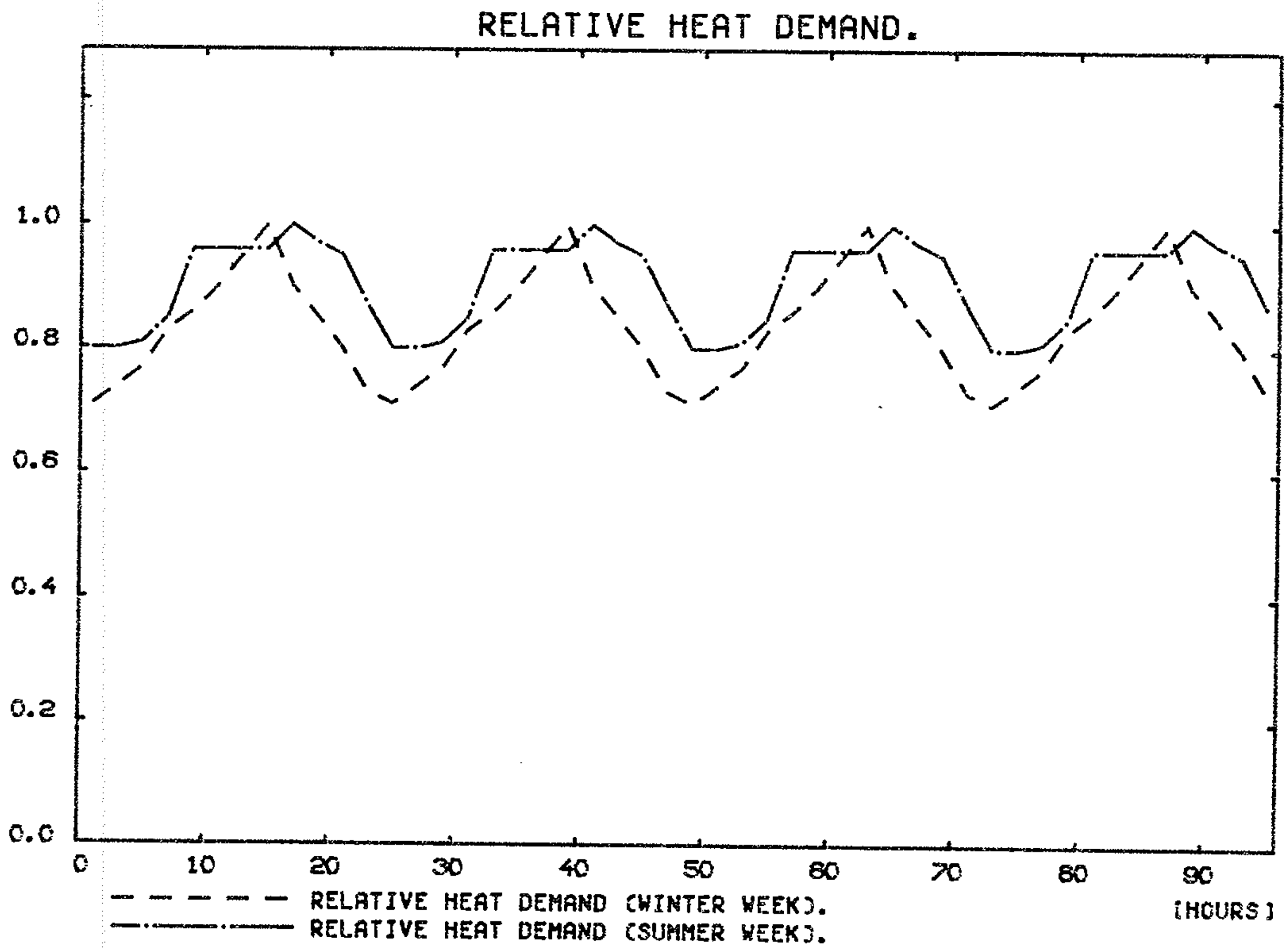

Fig. 6.9. Relative heat demand load profiles.

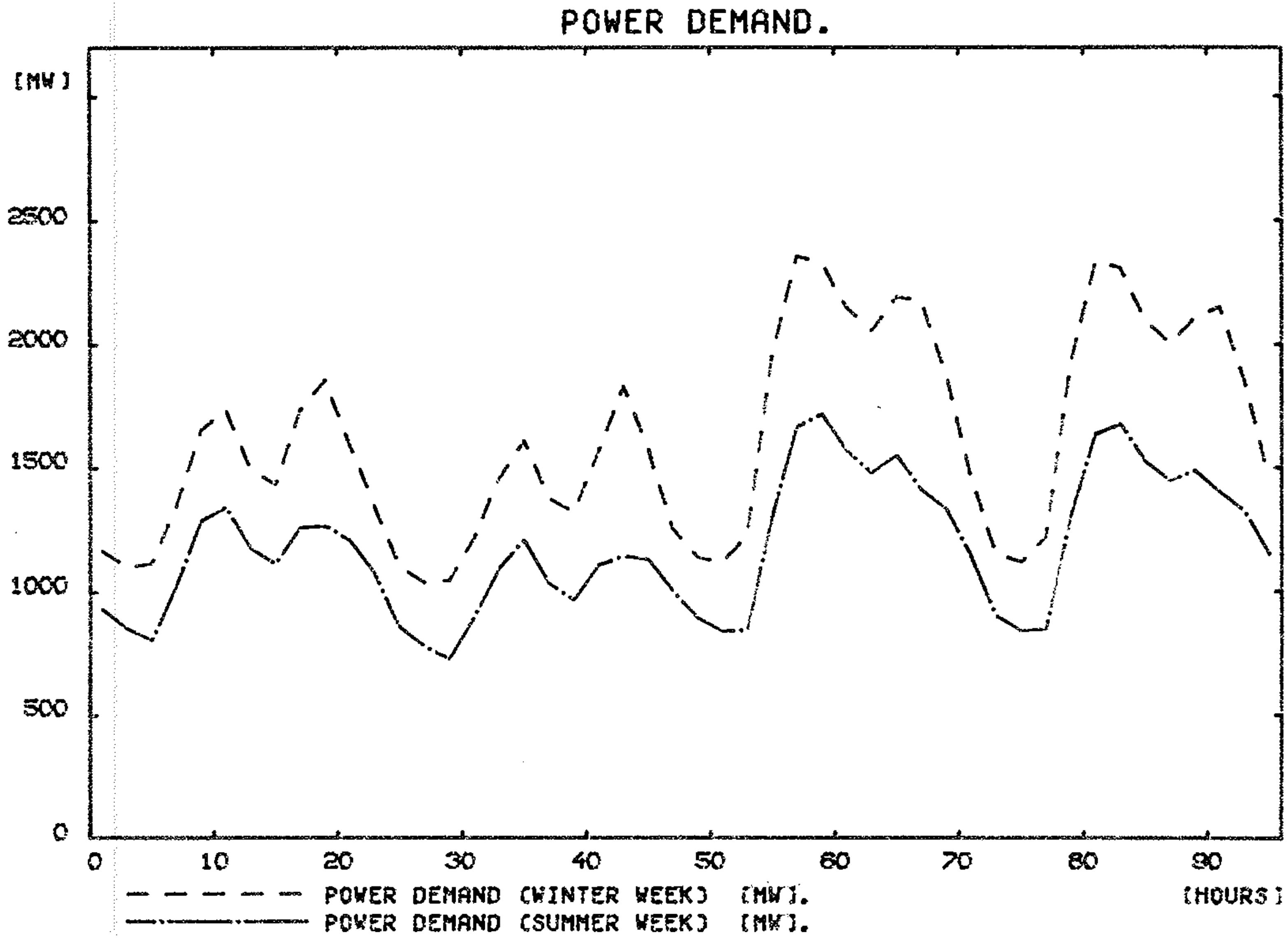

Fig. 6.10. Power demand.

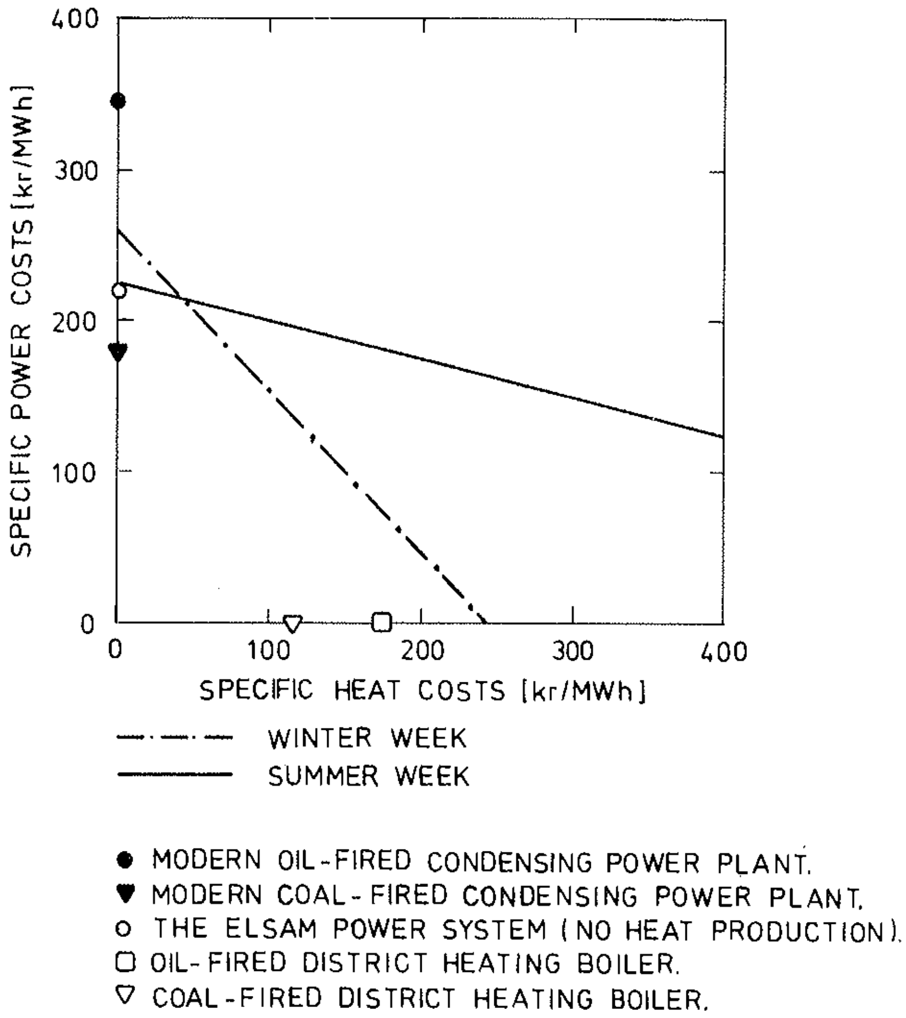

Fig. 6.11. Specific production costs.

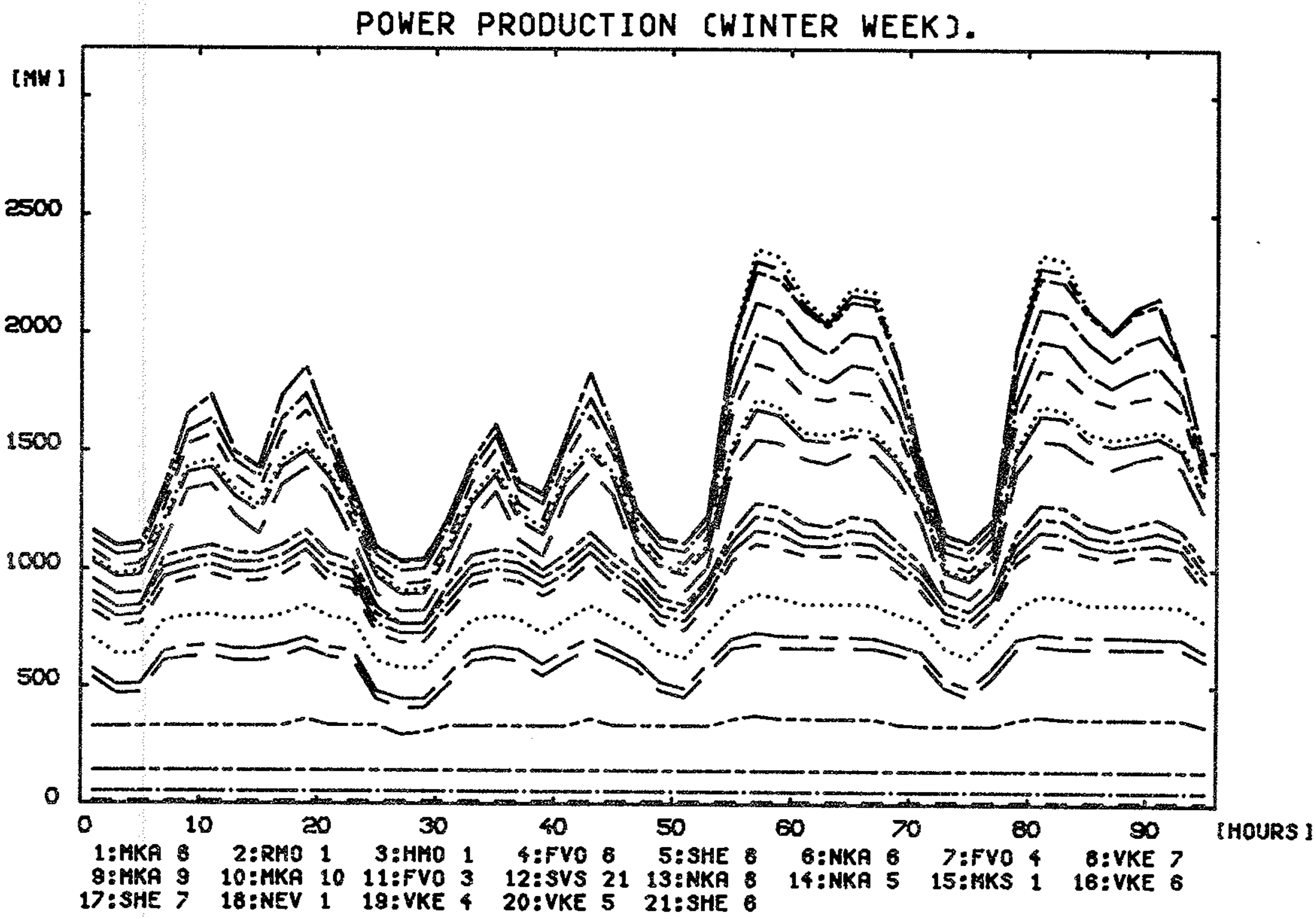

Fig. 6.12. Power production of individual plants (winter week). The numbering starts from the bottom of the figure.

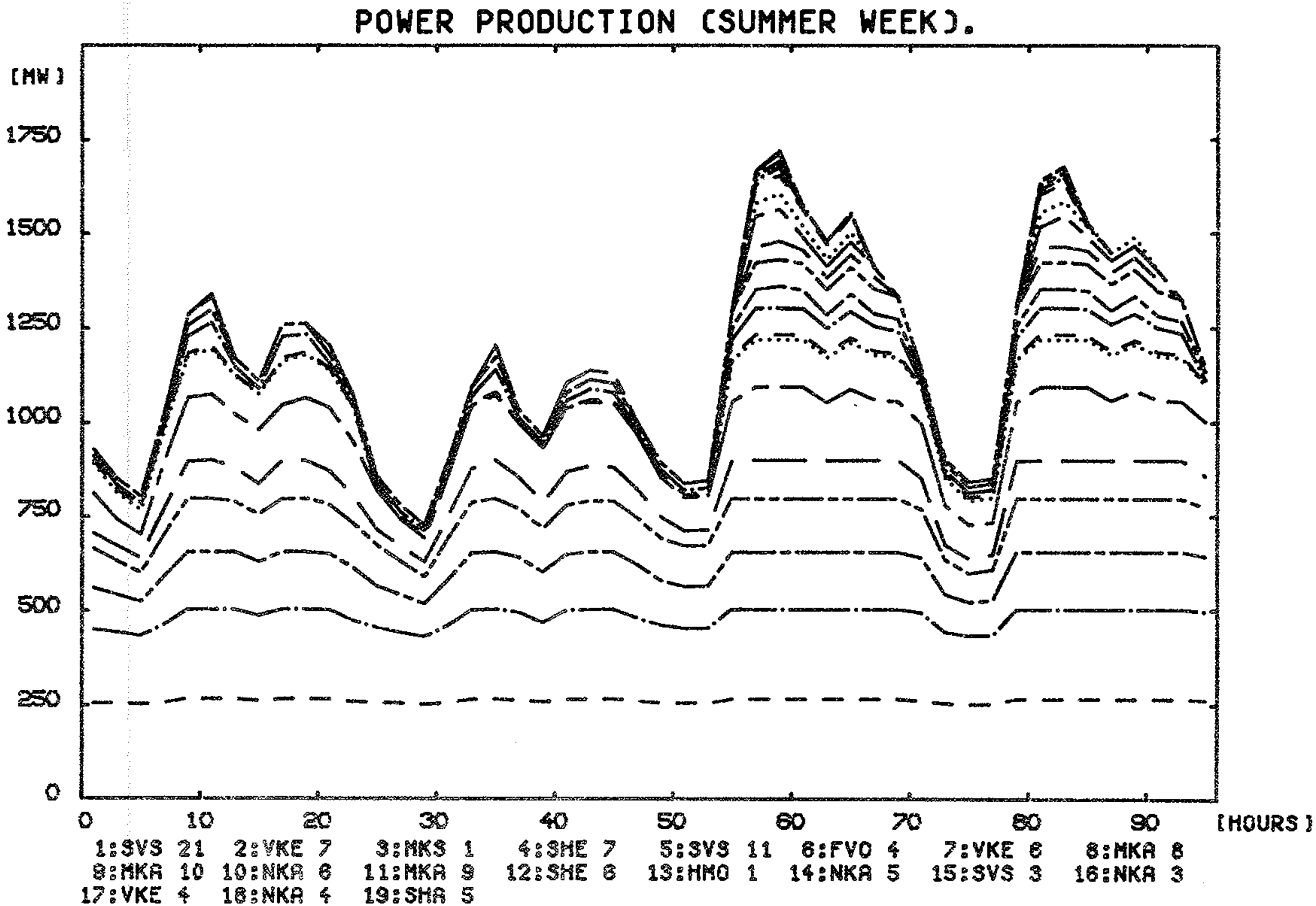

Fig. 6.13. Power production of individual plants (summer week).
The numbering starts from the bottom of the figure.

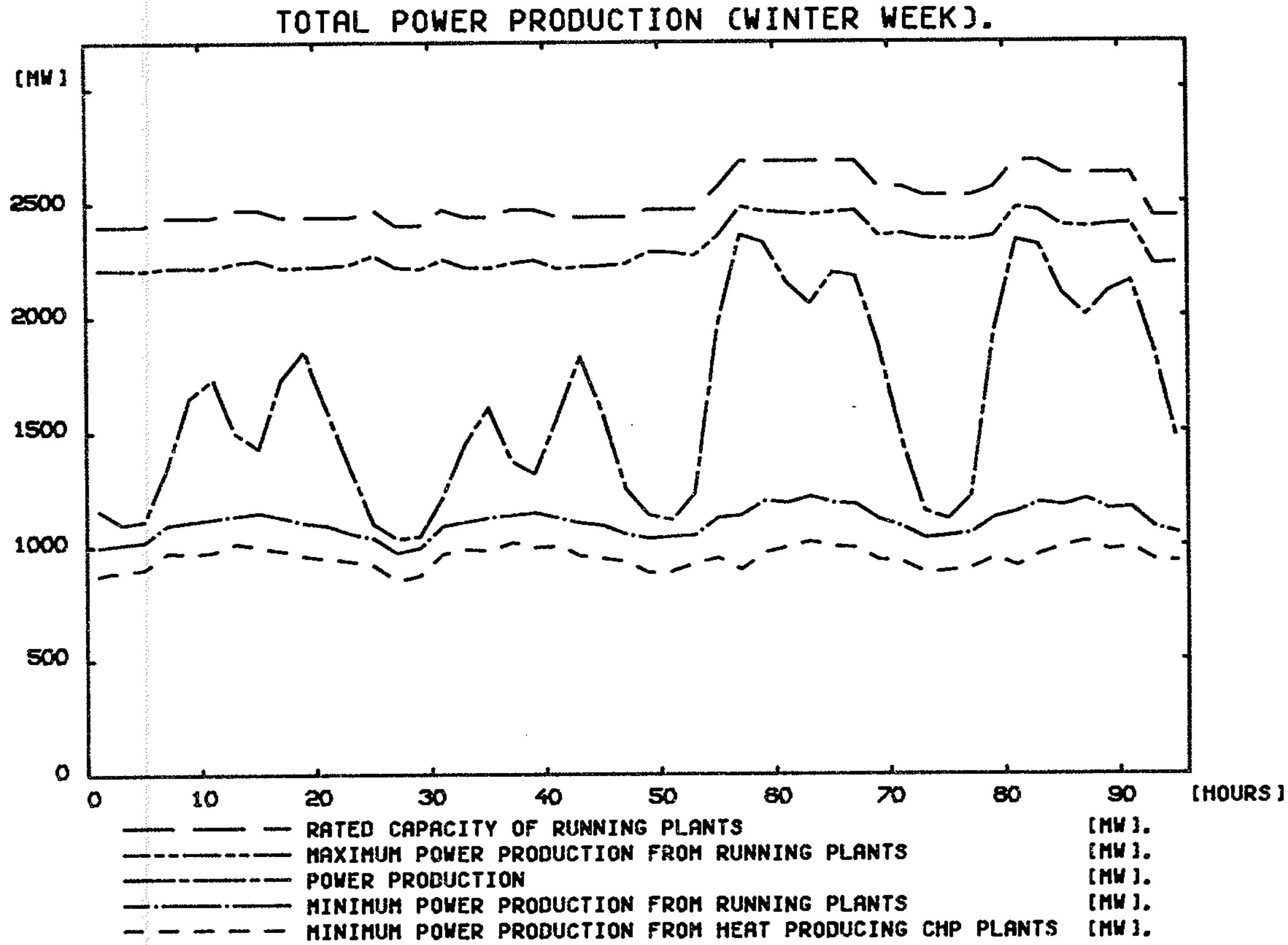

Fig. 6.14. Total power production (winter week).

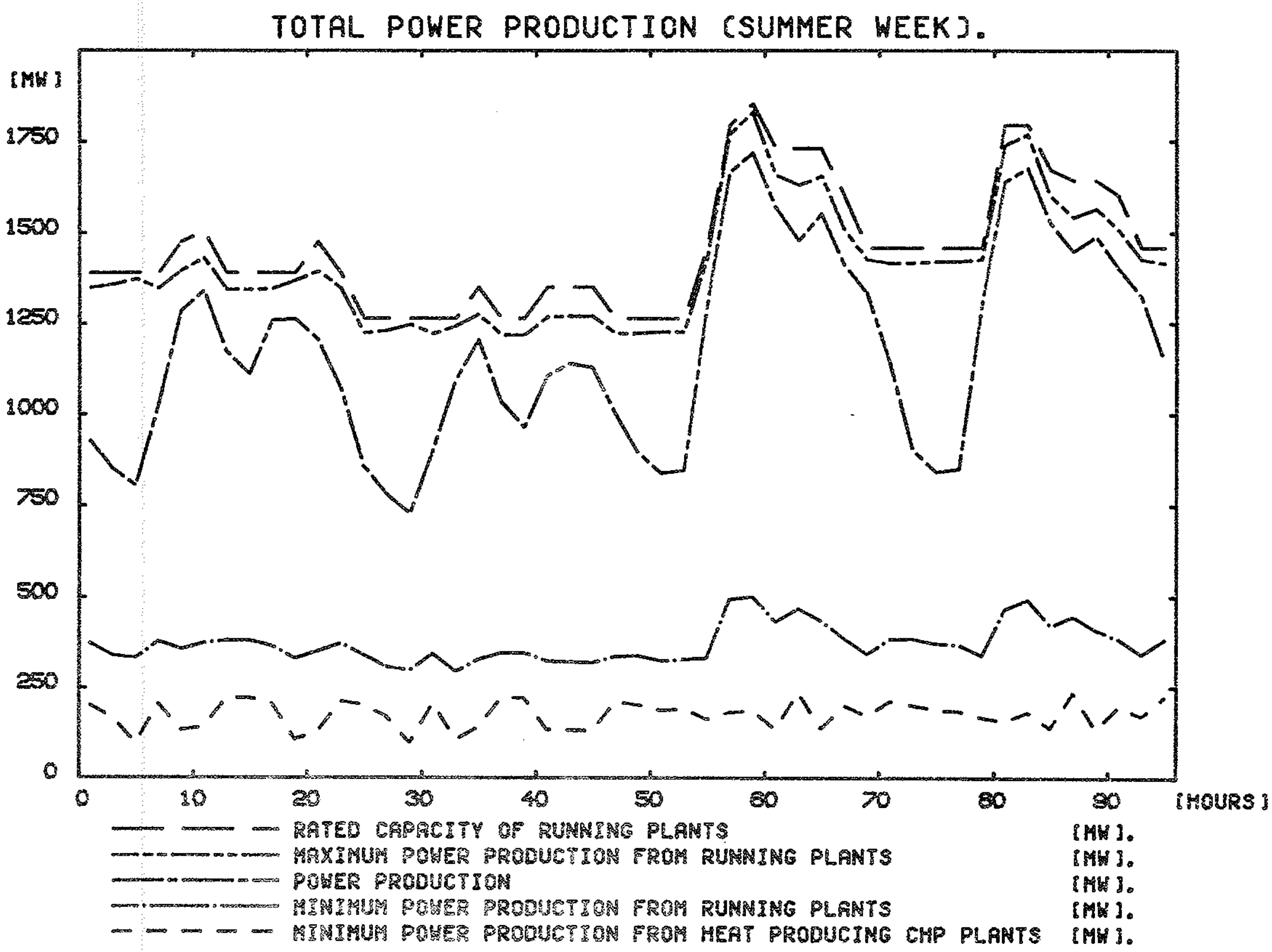

Fig. 6.15. Total power production (summer week).

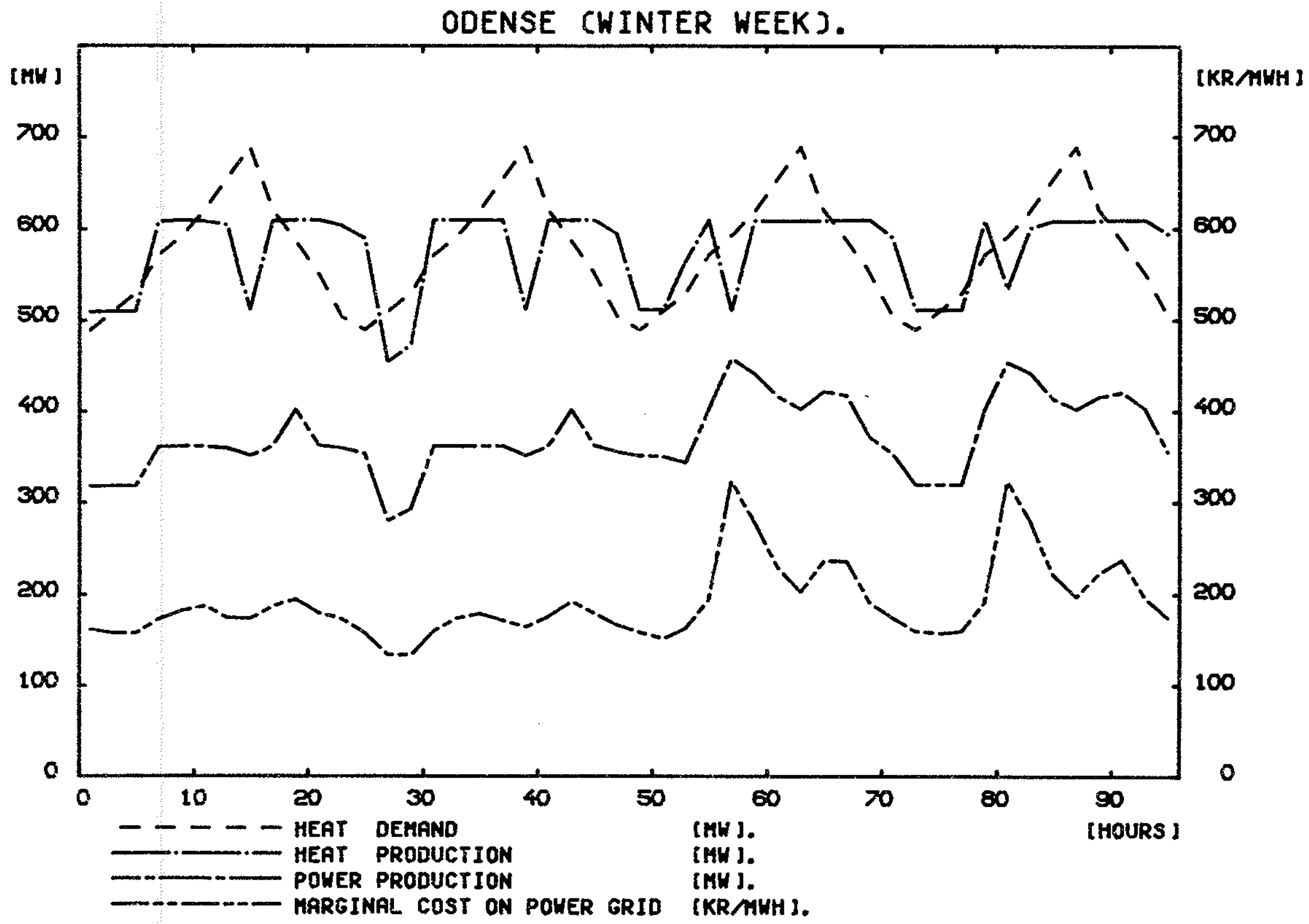

Fig. 6.16. Heat and power production in Odense (winter week).

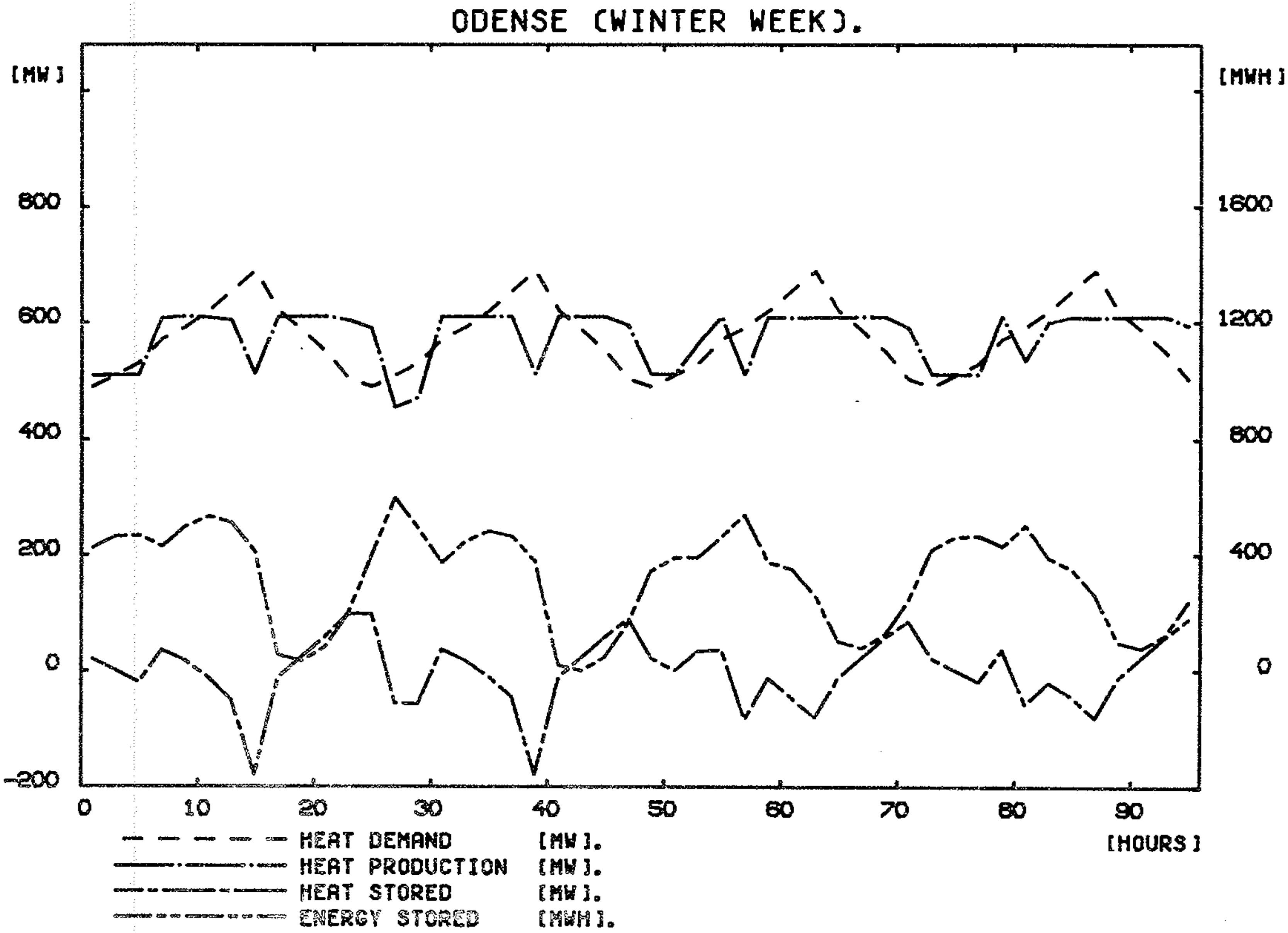

Fig. 6.17. Operation of heat storage facility in Odense (winter week).

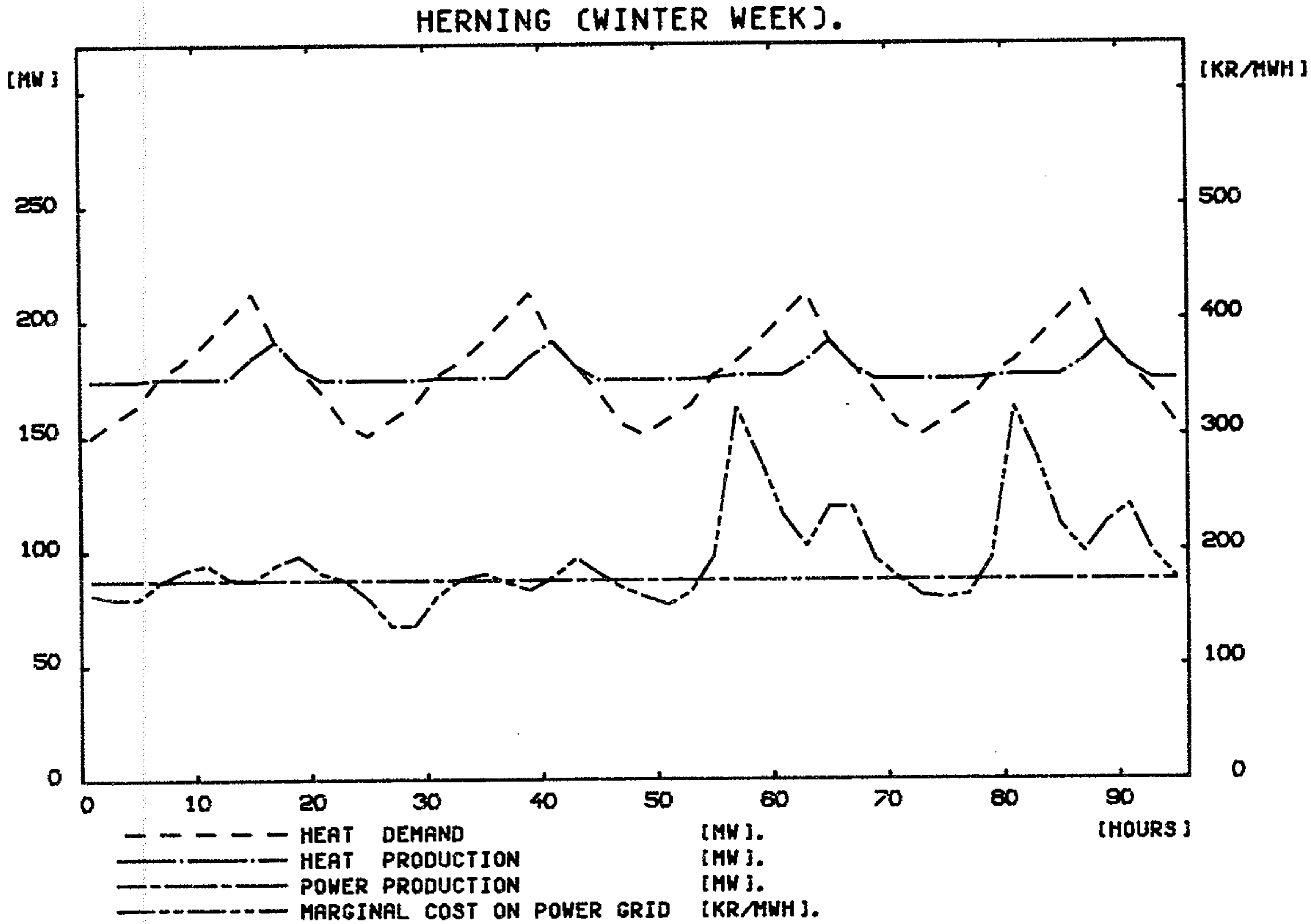

Fig. 6.18. Heat and power production in Herning (winter week).

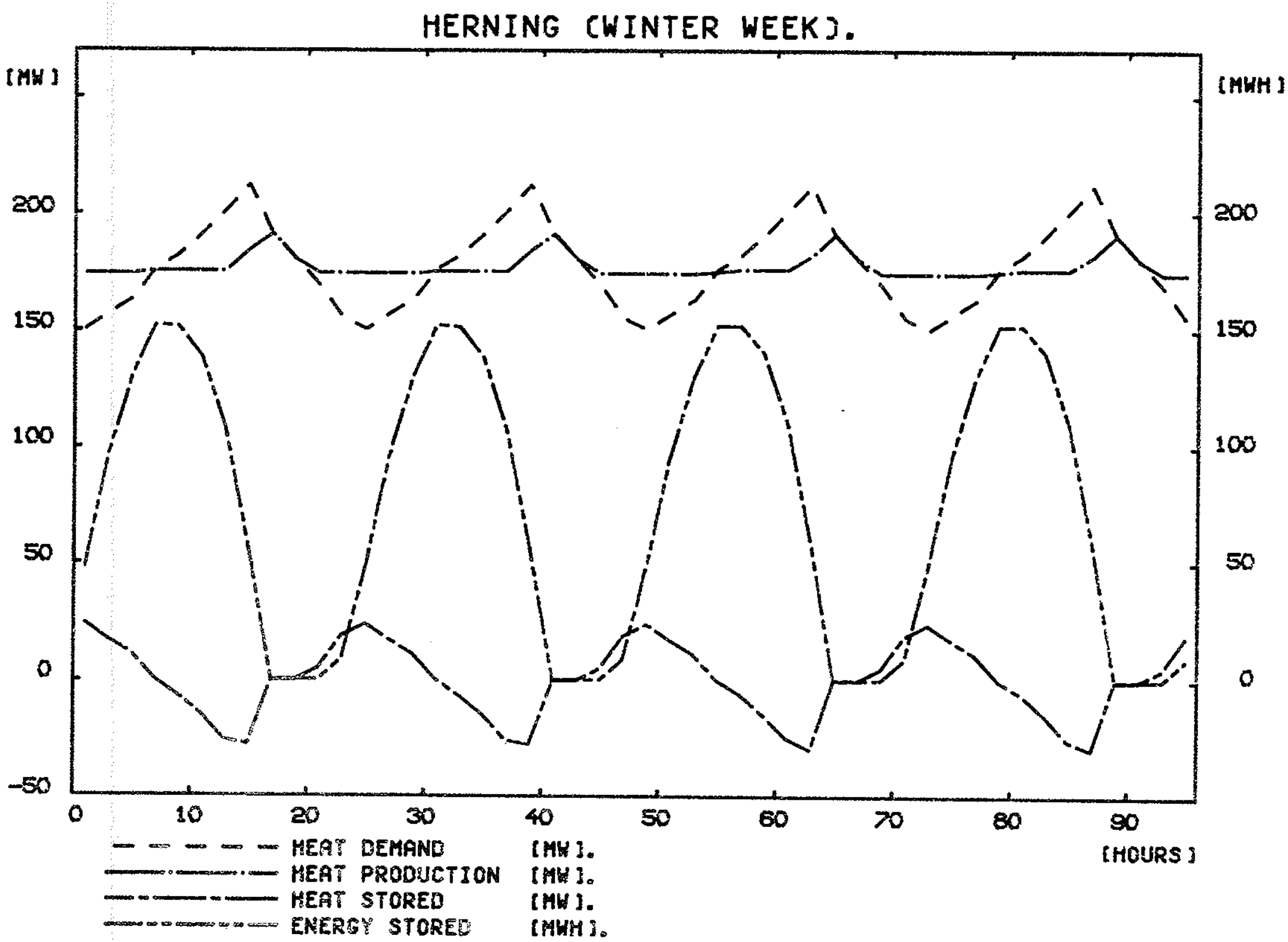

Fig. 6.19. Operation of heat storage facility in Herning (winter week).

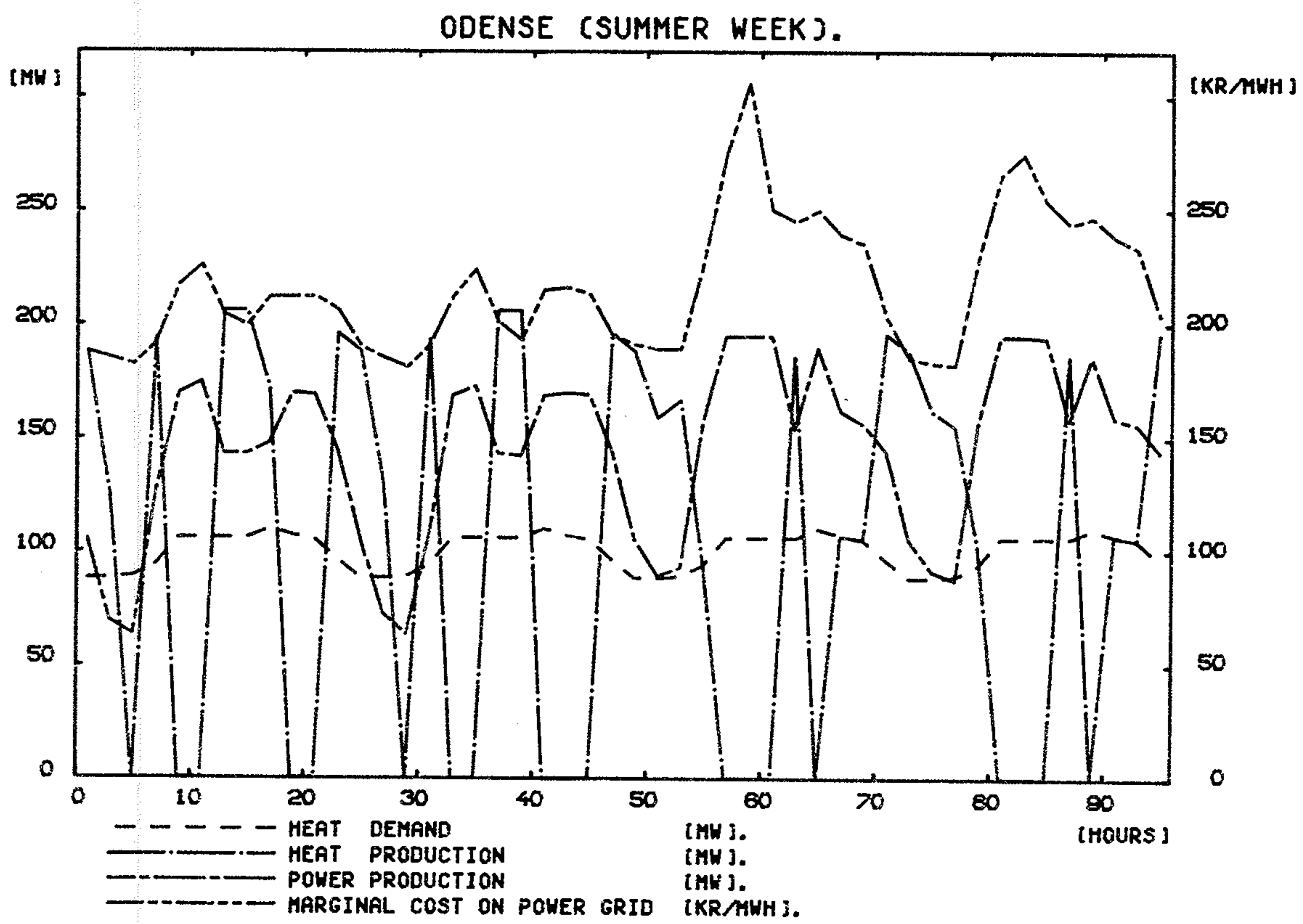

Fig. 6.20. Heat and power production in Odense (summer week).

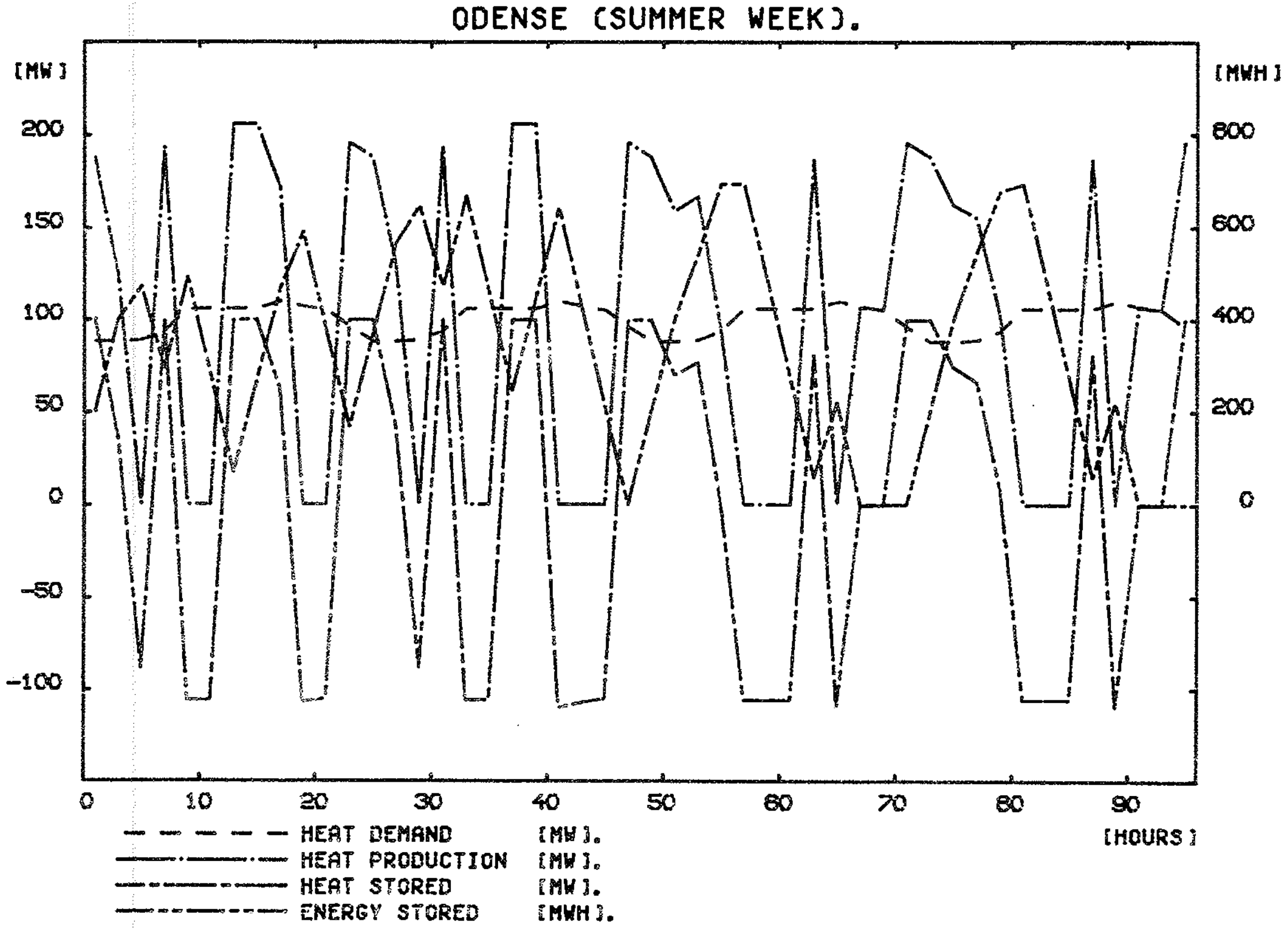

Fig. 6.21. Operation of heat storage facility in Odense (summer week).

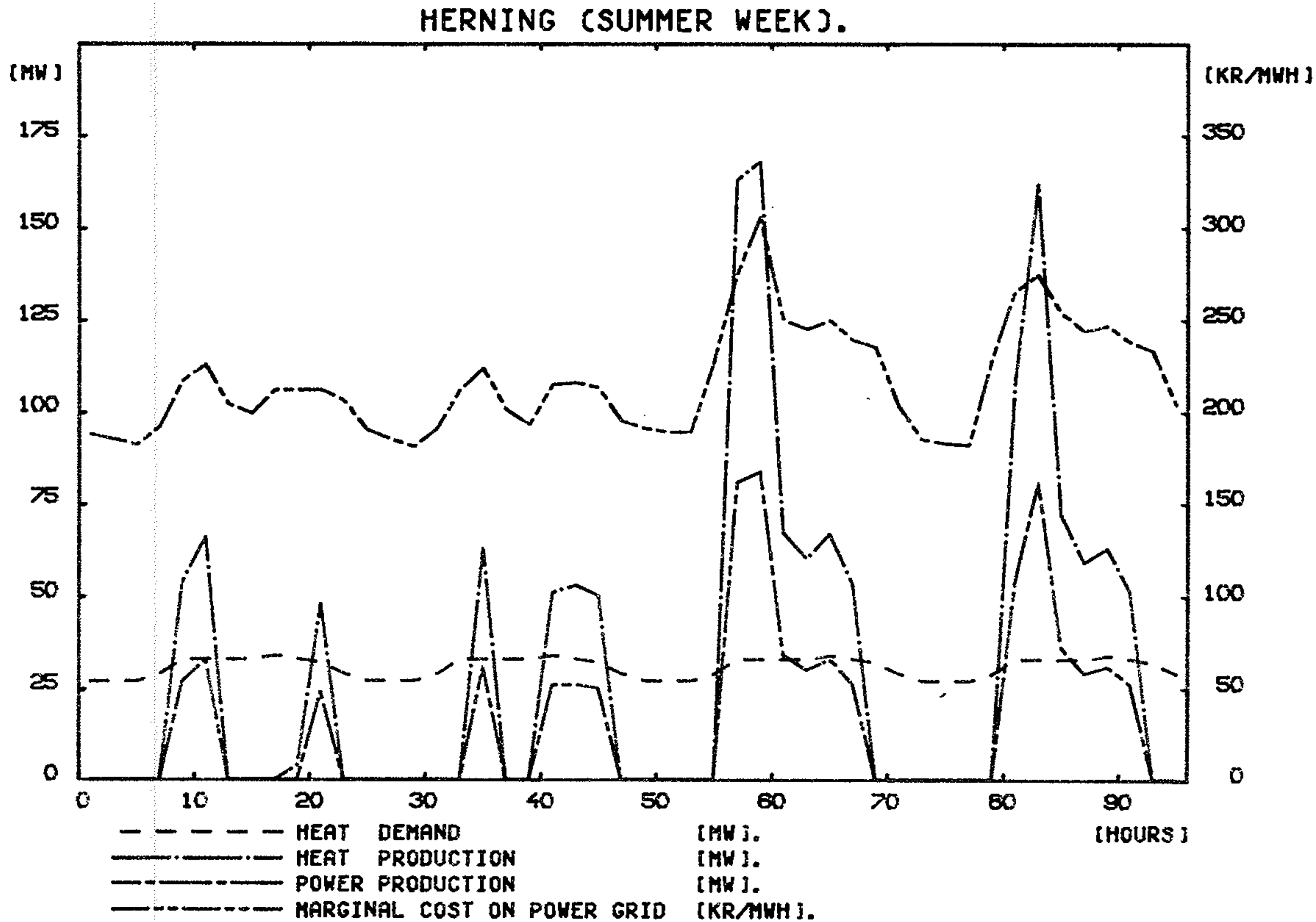

Fig. 6.22. Heat and power production in Herning (summer week).

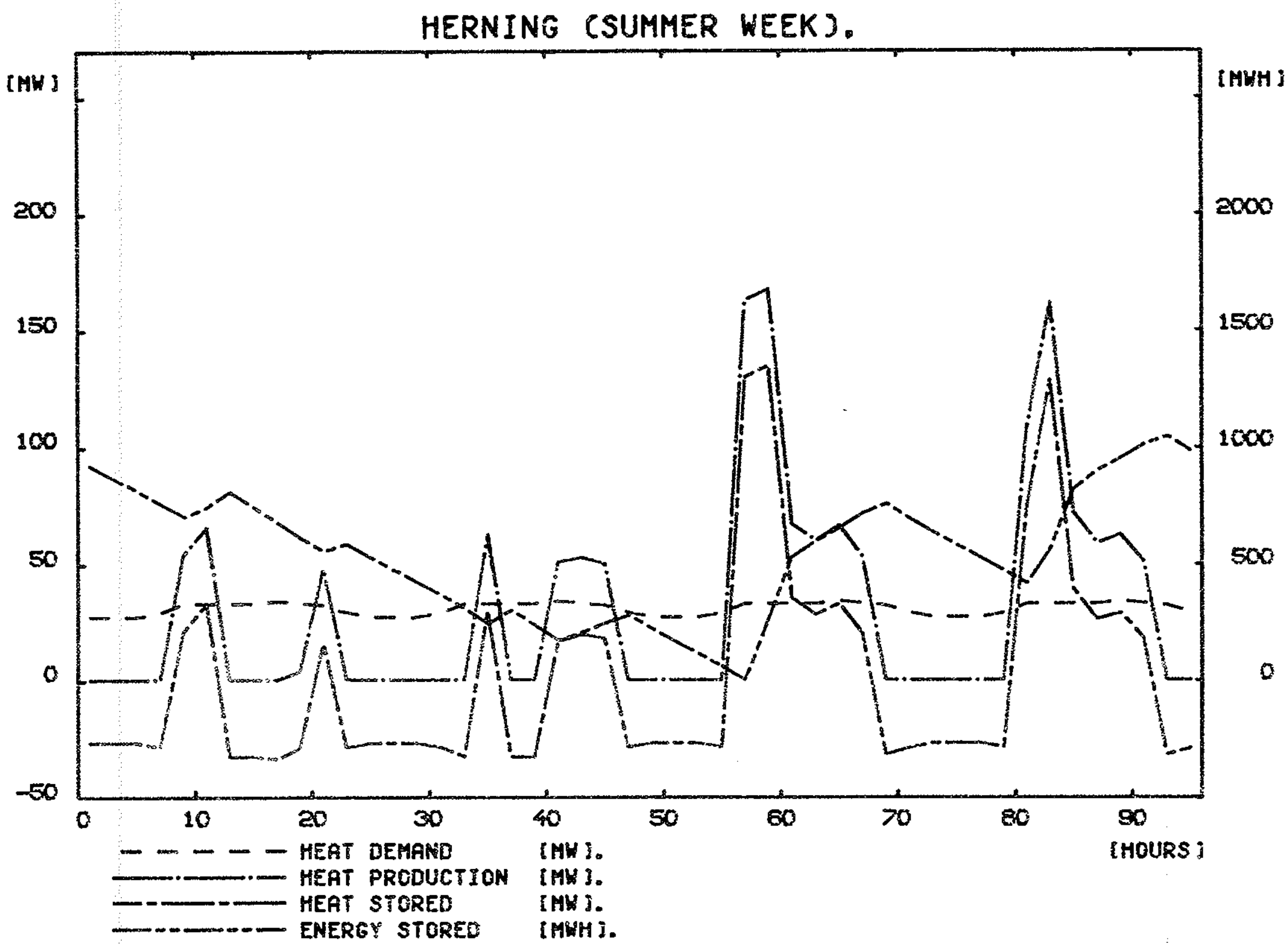

Fig. 6.23. Operation of heat storage facility in Herning(summer week).

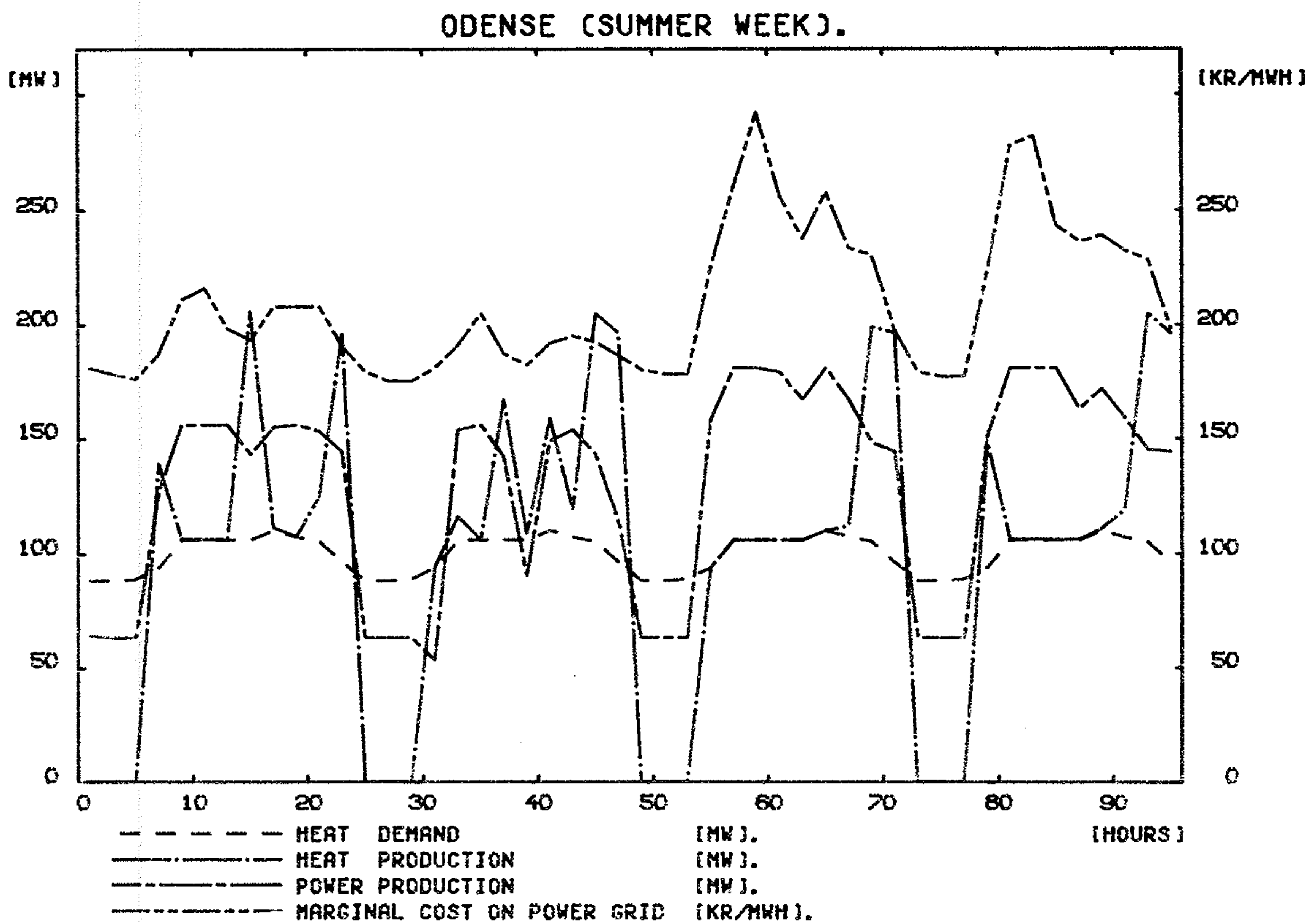

Fig. 6.24. Heat and power production in Odense (summer week).
Costs for operating the heat storage facility are introduced.

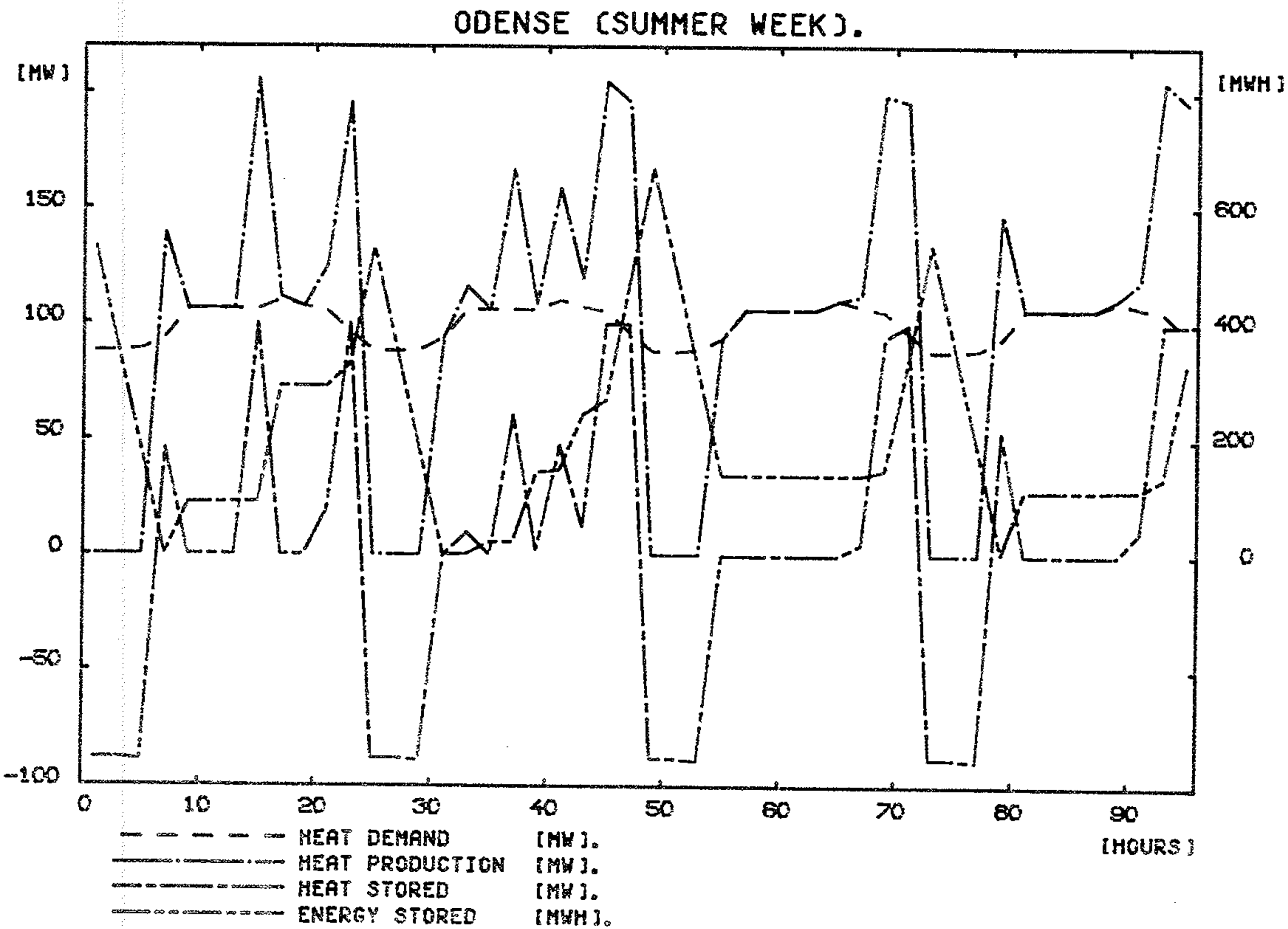

Fig. 6.25. Operation of heat storage facility in Odense (summer week). Costs for operating the heat storage facility are introduced.

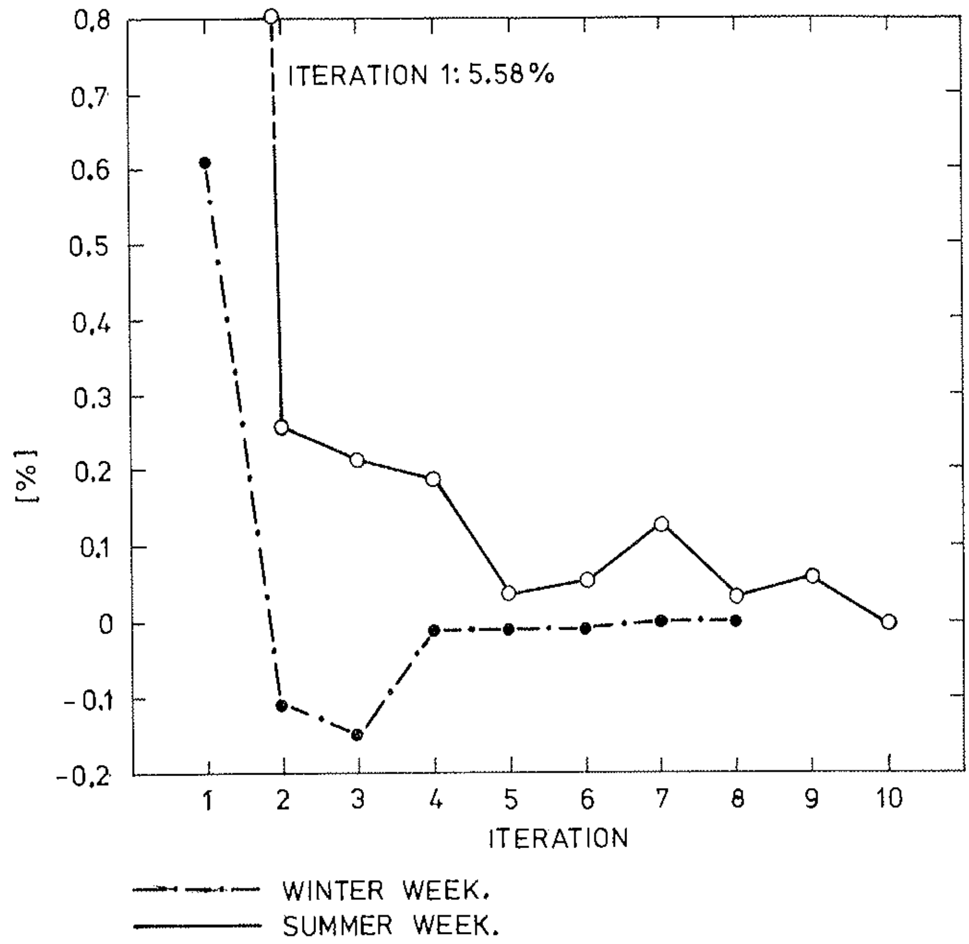

Fig. 6.26. Total costs for each iteration around the heat and power production parts of the model. Deviation from costs in last iteration.

APPENDIX A. THE BRANCH-AND-BOUND METHOD

A.1. Introduction

Production scheduling as formulated in Section 4.2 is an optimization problem involving both zero-one and continuous optimization varables. The formulation allows for a partial separation of the integer and continuous variables: The cost function C is easily separated into running costs $C_{tot}(\underline{\underline{P}})$ which do not depend on the U's, and start-up costs $C^{s}_{tot}(\underline{\underline{U}})$ which do not depend on the P's. Table 4.1 shows that the only constraint that links together the U's and the P's is constraint number 7.

This partial separation of the variables makes it possible to consider the optimization problem as an integer programming problem with an associated continuous optimization sub-problem: If a solution (perhaps not the optimal one) for the unit commitment problem obeys all constraints (except numbers 7 and 8) the start-up costs given by the unit commitment matrix $\underline{\underline{U}}$ can be found. Then the continuous sub-problem (i.e. the load dispatching) can be solved to find the minimum running costs. Table 4.1 shows that the only constraints to be observed by the load dispatching are number 7 and 8. As constraints 1 and 2 are obeyed by the U's, we are ensured that a feasible solution to the continuous sub-problem exists.

The integer programming problem can be solved by the Branch-and-Bound approach. In the remaining part of this appendix the Branch-and-Bound method will be considered from a general point of view. In its limitations the discussion is aimed at the application to the unit commitment as described in Sections 4.3.2 and 4.3.3. The load dispatching is dealt with in Sections 4.3.4 and 4.3.5.

A.2. General discussion of the Branch-and-Bound method

The general optimization problem dealt with in this section has only zero-one variables (i.e. no continuous variables). It can be formulated in the following way:

$$\begin{array}{lll} \text{Minimize } f(\underline{x}) & & \\ \text{subject to } g_k(\underline{x}) = 0 \quad , & & k=1,2, \ldots, m_{eq} \\ \text{and to} \quad g_k(\underline{x}) \geq 0 \quad , & & k=m_{eq}+1,\ m_{eq}+2, \ldots, m \end{array}$$

The costs to be minimized depend on the zero-one variables (the optimization variables) x_i, $i=1,2, \ldots, n$ in a way given by the cost function $f(\underline{x})$. The solution space is limited by the m constraints. The constraints define a set of feasible solutions to the problem. The task then is to find the optimum solution within this set. The functions $f(\underline{x})$ and $g_k(\underline{x})$ are assumed to be nonlinear.

There are 2^n combinations of the n zero-one variables as shown in the decision tree in Fig. A.1. A zero-one variable not

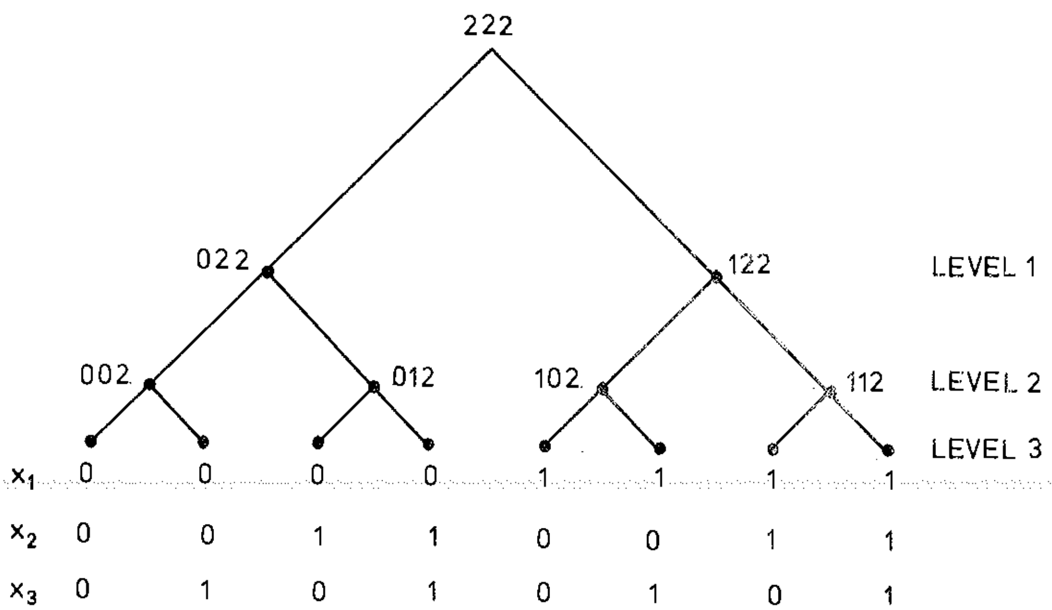

Fig. A.1 Decision tree with three zero-one variables. $x_i=2$: No decision is yet taken for x_i.

yet fixed to zero or one is given the value two. Therefore, a statement such as "$x_1 = 2$" should be interpreted as "$x_1 = 0$ or $x_1 = 1$". The decision tree consists of nodes and branches. At the top node no variables are fixed, i.e. they are all written as 2. A set of feasible solutions obeying the constraints is associated with each node. This set is partitioned into two disjunct subsets by fixing a variable to zero or one. When this decision is taken one moves down one of the two descending branches to arrive at a node in the next level of the tree. The set of feasible solutions associated with some node might be empty.

In an integer (zero-one) programming problem of reasonable size the number of combinations of the variables (at the bottom level of the decision tree) is very large. If, for instance, there are 30 variables, they can be combined in $2^{30} \approx 10^9$ different ways. It is obvious that even for modern fast computers it is impossible to evaluate all these solutions: For a specific combination of the variables it should first be checked whether or not the constraints are obeyed, and if they are the costs should be found. As described below, the Branch-and-Bound technique makes it possible to discard large subsets at the same time without having to consider all the individual solutions contained in the subset. The strength of the Branch-and-Bound technique lies in this deletion of whole sets of solutions without having to consider the individual members.

As the Branch-and-Bound technique is implemented for the unit commitment the computations start at the top of the decision tree where no integer variables are yet fixed. From this node a path is followed down the tree corresponding to moving from level to level until finally the last level at the bottom (level n) is reached where all variables are fixed to zero or one. Such a path is not followed from the top to the bottom without interruptions. At each of the intervening nodes a certain procedure is followed which might result in a complete deletion of the set of solutions associated with the node (the node is said to be closed). The procedure also might leave the node open for a while and jump to another open node in another part of the tree. The exact procedures followed are described in Section 4.3.2.

Let us now assume that we have arrived at some node in the decision tree, and that this node is not at the last level. The first thing to do is check whether the set of feasible solutions associated with the node is empty or not. Since the constraints might have a very complicated structure this is an almost impossible task. The problems arise because the constraints define certain combinations of values (zero, one) for the optimization variables as feasible, and others as unfeasible. But at the node considered there might be many variables that have not yet been fixed to zero or one. Therefore, it is very difficult to find whether, among those many solutions that can be made from the node considered, there exists at least one that obeys all the constraints.

For the reasons just mentioned a check is not made as to whether the set of feasible solutions is empty or not. Instead a distinction between the following two cases is made:

1) The set of feasible solutions associated with the node is empty.
2) The set of feasible solutions associated with the node might be empty or not.

The introduction of this procedure is accomplished by replacing each of the constraints by a less restrictive one (in the following called a "soft" constraint). These new soft constraints are constructed so that every solution that violates them will also violate the original "hard" constraints. On the one hand, the soft constraints must be as close to the hard ones as possible, but on the other, it should not take too much computer time to check them. It should be noticed that the introduction of soft constraints does not change the definition of the optimization problem considered: The correct hard constraints will still be used at the last level of the decision tree where all variables are fixed to one or zero.

If the set of feasible solutions associated with the node considered is empty (case 1 above) the node is closed and not considered further. This means that the node in question and all the descendant nodes including those at the last level are deleted.

If the soft constraints cannot tell whether the set of solutions associated with the node is empty or not (case 2 above) a common underestimate of the costs for all feasible solutions in the set is made. This estimate should be as big as possible, but on the other hand it must be a true underestimate, i.e. it must be smaller than the costs of any feasible solution in the set. In this way the underestimate constitutes a lower bound on the costs for the subset of solutions.

The cost of the best (i.e. cheapest) member of the set of solutions associated with a node will always be smaller than or equal to the cost of the best member of the subset of solutions associated with one of its two descendants. This follows because no new solutions are introduced by partitioning a set of solutions into two disjunct subsets. Therefore, the procedure for finding an underestimate of the costs should be defined so that this underestimate will never decrease from a node to its descendants.

If the lower bound for the node considered is larger than the upper bound (defined below) on the optimum solution it is obvious that the optimum cannot be found by partitioning of the set. Therefore the node is closed (discarded).

Thus it is seen that a node can be closed for two reasons: Either because it is violating the constraints or because it can be estimated that all its descendants in the bottom level of the decision tree are too expensive.

If the node is not violating the soft constraints, and if its lower bound is not too big, it should be further partitioned by fixing one more variable to zero or one. This might take place immediately, or the node might be stored as an open node for later computations. In the meantime other nodes are considered.

If a path down the decision tree is not stopped at some closed node it will finally end at the lowest level at the bottom of the tree where all variables are fixed to zero or one. The computations performed for nodes at this level differ somewhat from those performed for the above levels.

For a given solution at the bottom of the decision tree the observance of each constraint is checked. Here the original "hard" constraints, and not the "soft" ones, are used. If one (or more) more) of the constraints is violated the node is closed, and nothing more is done concerning this node. If all constraints are obeyed, the solution is said to be feasible, and the costs associated with it are found.

During the computations the best (i.e. cheapest) hitherto found solution is stored together with its costs. When a new feasible solution is found, the costs associated with this are compared with the costs of the hitherto best one. If the new solution is more expensive it is discarded. If, on the other hand, it is cheaper, it will replace the hitherto best solution.

The cost of the best hitherto found solution form an upper bound on the costs of the optimum solution. As mentioned above this upper bound is used in a comparison with the lower bounds associated with the individual nodes of the decision tree. Each time the upper bound is reduced because a new and better solution is found, the lower bounds of all open nodes of the decision tree are compared with this reduced upper bound (see Fig. A.2). If a lower bound is found to be larger than the upper bound, the node in question is discarded and not considered further. Therefore an improved upper bound can drastically reduce the number of nodes left open for later investigations.

When there are no more open nodes the computations can stop. The optimum solution is then identical to the solution that forms the last updated upper bound.

The Branch-and-Bound method discussed above will find the solution that has the minimum costs. If it is satisfactory to get a solution that is known to be no more than say 0.1% more expensive than the cheapest one, the concept of ε-optimality can be introduced. This is simply done by multiplying the upper bound by $1/(1+\varepsilon)$ before it is compared with the lower bounds. This will cause an increased number of nodes to be closed, thus saving computer time.

From the above discussion of the Branch-and-Bound method it can be seen that the order in which the integer variables are fixed will influence the number of nodes that have to be considered before the optimum is finally found. Therefore, the variables that are supposed to have the greatest influence on the costs should be fixed at the first levels of the decision tree. By doing so, many nodes can be expected to be discarded already at the upper levels of the tree.

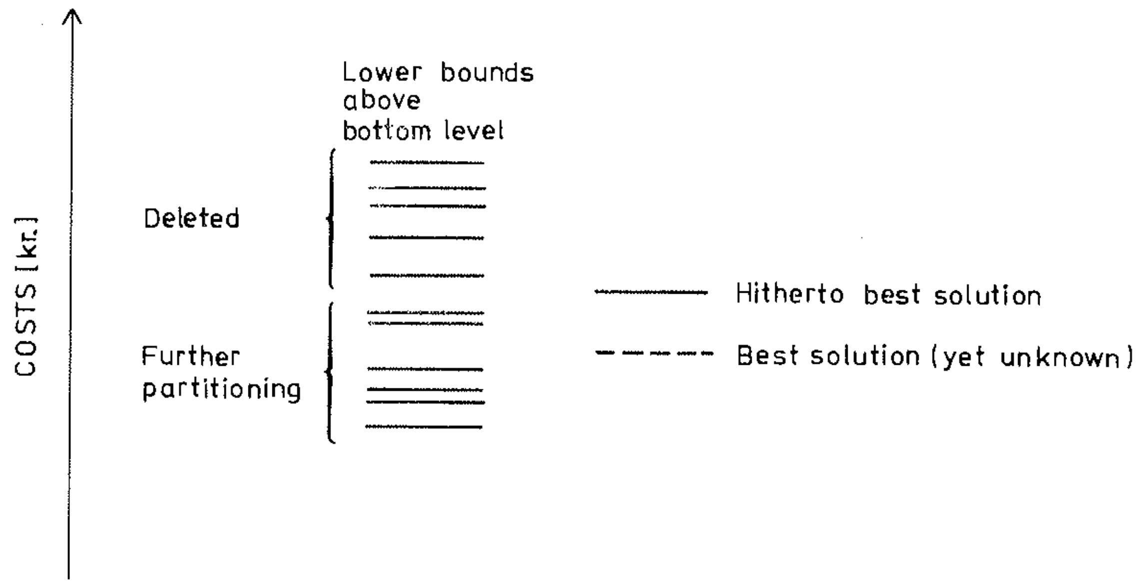

Fig. A.2. Illustration of the deletion of subsets with lower bounds greater than the costs of the hitherto best solution.

When the computations associated with a certain node of the decision tree are finished, one needs a strategy to determine which of the open nodes should be considered next. Likewise, when the set of solutions associated with a node is partitioned by fixing the appropriate integer variable to zero and one, it must be decided which of the two subsets to turn to, i.e. which of the two succeeding nodes should be considered, and which should be left open for later investigations. The answers to these two questions are specific to the optimization problem and will therefore be dealt with in Section 4.3.2.

APPENDIX B. LIST OF VARIABLES.

In this appendix the variables used in the report are given together with a short description. Moreover reference is made to the section where they are defined. Subscripts t and i (time and plant indices) are omitted where possible.

Variable	Unit	Description	Defined in Section
N_g		Number of power plants	4.2.1
N_h		Number of heat producing plants	5.4.2
N_t		Number of time steps	4.2.1
i, j		Plant indices	4.2.1
t		Time index	4.2.1
Δt	h	Length of time step	4.2.1
U, U_{it}		Unit commitment variable	4.2.1
$\underline{\underline{U}}$		Unit commitment matrix. $\underline{\underline{U}}=\{U_{it}\}$	4.2.1
U_i		The i'th row of U	4.2.1
P, P_{it}	MW	Power production	4.2.1
$\underline{\underline{P}}$	MW	Power production matrix. $\underline{\underline{P}}=\{P_{it}\}$	4.2.1
P_o, P_{oit}	MW	Equivalent power production. $P_o=P+c_v \cdot H$	5.2.3
H, H_{it}	MW	Heat production	5.2.2
H_S, H_{St}	MW	Heat delivered to storage facility	5.4.2
E_S, E_{St}	MWh	Heat contents of storage facility	5.4.2
P_i^{min}, P_i^{max}	MW	Bounds on P_{it}	4.2.2
P_{oi}^{min}, P_{oi}^{max}	MW	Bounds on P_{oit}. $P_{oi}^{min}=P_i^{min}$ and $P_{oi}^{min}=P_i^{max}$	5.2.3
H_i^{min}, H_i^{max}	MW	Bounds on H_{it}	5.4.2
H_S^{min}, H_S^{max}	MW	Bounds on H_{St}	5.4.2
E_S^{min}, E_S^{max}	MWh	Bounds on E_{St}	5.4.2
P_D, P_{Dt}	MW	Power demand	4.2.2
P_L, P_{Lt}	MW	Power transmission losses	2.8
P_L', P_{Lt}'	MW	Simplified power transmission losses	4.2.2
P_L'', P_{Lt}''	MW	Simplified power transmission losses	4.2.2

Variable	Unit	Description	Defined in Section
P_L^{LB}, P_{Lt}^{LB}	MW	Simplified power transmission losses	4.3.5
P_R, P_{Rt}	MW	Spinning reserve	4.2.2
H_D, H_{Dt}	MW	Heat demand	5.4.2
H_D', H_{Dt}'	MW	Apparent heat demand. $H_D'=H_D+\eta_S \cdot H_S$	5.4.2
H_L, H_{Lt}	MW	Heat transmission losses	5.4.2
c_m, c_{mi}	MW/MW	Back-pressure constant	2.3
c_v, c_{vi}	MW/MW	Slope of equi-fuel-consumption lines for extraction power plants	2.4
η, η_i		Effectivity (1-losses) in heat transmission	5.3.2
η_S		Effectivity (1-losses) in heat transmission from storage facility	5.4.2
B_{ij}	MW^{-1}	B-constants giving the power transmission losses	2.8
B_{oi}	MW/MW		2.8
B_{oo}	MW		2.8
ITL, ITL_i	MW/MW	Incremental power transmission losses	4.3.4
λ	kr/MWh	Marginal power production costs	4.3.4
λ_h	kr/MWh	Marginal heat production costs	5.3.2
C_p	kr/MWh	Selling price of co-produced electricity	5.2.3
C_S	kr/MWh	Operating costs of storage facility	5.4.2
$C_i(P_{it})$	kr/h	Running costs	4.2.1
$C_i^S(\underline{U}_i)$	kr	Start-up costs	4.2.1
$C_{tot}(\underline{\underline{P}})$	kr	Total running costs	4.2.1
$C_{tot}^S(\underline{\underline{U}})$	kr	Total start-up costs	4.2.1
C	kr	Total production costs	4.2.1
$C_{tot,t}(H_{Dt}')$	kr/h	Total heat production costs	5.4.2
$C_{marg,t}(H_{Dt}')$	kr/MWh	Marginal heat production costs	5.4.2

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